AN INVESTIGATION OF MULTIPLE-ITEM ORDERING FROM A COMMON SOURCE

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AN INVESTIGATION OF MULTIPLE-ITEM

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SUMMARY

Managerial systems for inventory control need to recognize that inventories are usually embedded in large systems and that often non-inventory considerations may enter or even dominate inventory policies. One of the non-inventory considerations of interest is the source of supply. Among the prospective suppliers to be favored with a firm's business, each can typically be the source for several items.

This thesis is concerned with the development of a basic procedure to determine multiple item ordering quantities from single suppliers. One system of interest is the fixed reorder cycle system, where it is possible to group orders for a number of individual items from one supplier and realize cost savings. An analytical method developed by Brown (1) was investigated and its use was extended for establishing least cost multi-item procurement practices in terms of the order cycle length and the items to be ordered at the beginning of each cycle.

The parameters of the system such as demand and lead time are treated as deterministic and stochastic variables respectively. A sensitivity analysis was performed on the cost parameters of the inventory system to establish the importance of estimation errors. The economic consequences of quantity discounts were analyzed, and expressions were developed for determining the safety stocks, average working stocks, and average inventory on hand plus on order. The inventory policy was tested with an illustration problem and the model was exercised using actual data. A comparison was made with a procurement

inventory policy encountered in practice. The results showed considerable improvements for the joint ordering inventory model.

CHAPTER I

INTRODUCTION

In inventory system management an effective strategy may call for the use of multiple-item ordering from a common source of supply. This is a situation where the purchaser and suppliers benefit from some regular orders that include a wide range of items rather than frequent orders for a few needed items. For example, if a large number of items are regularly ordered from one supplier, it may be worthwhile to abandon an order point system and place orders for all items by some regular schedule in order to reduce total cost by ordering products with relatively small dollar value at the same time when the large dollar value items are ordered in addition. An order placed for multiple items from a single supplier may qualify for a quantity discount and may reduce both the shipping costs and the costs of procurement.

The problem addressed in this thesis is to select a least cost procurement strategy for multiple item ordering from different vendors. The major variables are the length of the reorder cycle for each vendor and the particular parts to be ordered in each cycle. The presumption is that each vendor will supply both high volume fast moving items and low volume slow moving items. The purchaser will probably wish to order high volume items in every cycle but may wish to order a slow moving item every second, third, or fourth cycle. An analytical procedure is developed and demonstrated for determining both the least cost cycle time and the order frequency for the different items in a family.

By using a single cycle time T, some items will be ordered at less frequent than optimal intervals, thus incurring excessive holding costs. Other items will be ordered at more frequent than optimal intervals. Still other items can be ordered at intervals 2T, 3T, and so forth. There is no requirement that every item be ordered at every interval. The requirement is that no item be ordered between intervals. The principal problem involves three steps: (1) identifying all members of each procurement family so that their lots can be ordered from one vendor at one time; (2) determine the length of the order cycle T, the average time between orders for this family; and (3) determine the order interval k_i, for each item in the family. The cycle multiple k_i, is the integer number of T length cycles between successive orders for item i. In general item i will be ordered once every k_i cycle in a k_iT-year supply. Table 1 shows the nomenclature used in the development of this system. While it is possible to determine optimal review periods for each item ordered, it may be advantageous to set a common review period for all items or for classifications of items in order to gain the advantages of grouping orders to common suppliers. Therefore, little attention is given to individual optimal review periods. Instead, review periods are set on the basis of ordering a family of items.

Nature of the Problem

In the specific case under study an analysis is made of a situation encountered in practice where a firm produces to meet a long-term contract. In this instance there is no problem in determining the market forecast for the finished goods, since it is fixed by a yearly contract. Finished goods demand data are stored in computer files and

Table 1. Nomenclature of System and Cost Parameters

Symbol	Description	Units of Measure
R_i	Annual requirements of item i	units/year
τ	Procurement lead time, generally a fraction of a year	year
C_i	Item cost, price of item i	$/unit
H	Holding rate, annual percentage of item cost	%/year
r_i	Carrying charge for item i	($/unit)/year
C_h	Holding cost	$/year
C_p	Procurement cost	$/year
q_i	Procurement quantity for item i	units
TC	Total yearly cost	$/year
T	Inventory cycle time, generally a fraction of a year	year
k_i	Multiple cycle time for item i	
SS	Safety stock level	units
NC	Number of cycles	
TOQ_j	Total family quantity ordered	units/cycle
ATOQ	Average total ordering quantity	units/cycle
D	Total demand of family items	units/year
AI	Average on hand inventory	units
MI	Maximum on hand inventory	units

production requirements from these files are exploded every month and compared against the current inventory status to develop detailed assemblies and purchased parts schedules. The production rate is known with certainty, thus the assembly lines are evenly loaded throughout the entire year. Therefore, the demand rates and requirements of all parts can be determined with great accuracy.

There is a considerable amount of purchasing of semi-processed materials, fabrication parts, components, maintenance, and operating supplies which tends to keep the proportion of the sales dollar expended for goods and services high. The supply of all these parts is so vital to the system, that efforts are made to diminish the possibilities of stockouts.

The purchasing function must be performed so as to minimize or eliminate disruptions in production resulting from the lack of any materials, equipment, or supplies, with a minimum investment in reserve inventories. For this reason it was decided not to expand the inventory system to all levels, but rather to concentrate efforts on developing the inventory system for only purchased parts.

Survey of the Literature

The focus of this thesis is on the investigation of techniques for ordering several items from a common source, and for basing the quantity ordered for any one item not only on its own stock status, but also on the stock status of all the items in the joint group. Much of the literature work done is related to production quantities for cycled items. Brown [1] formulated an analytical method which shows how the general concept behind the development of the economic order quantity can be extended to handle these families of items. Buffa and Taubert [3] developed a methodology where the production runs (lot sizes) of all products are determined jointly so that the scheduling interference is taken into account.

The development of the distribution of lead time demand is an essential step in the derivation of buffer stocks to absorb variations in demand and/or variations in supply lead time. A Monte Carlo development of lead time demand is described by McMillan and Gonzalez [14]. A numerical procedure to derive the joint distribution of lead time demand is illustrated by Fabrycky and Banks [7], which is applicable in those cases where demand has a Poisson, normal, or Chi-Square distribution. The distribution of lead time or Chi-Square distribution and the distribution of lead time need not conform to any specific form. The stockout acceptance factor is based upon the lead time demand distribution. Snyder [18] views this factor as an attempt to minimize the investment in inventory relative to an acceptable level of stockout previously specified by management.

The identification and measurement of costs is an important step in an inventory study as emphasized by Groff and Muth [8]. For some items, annual expenditure for costs other than inventory holding are influenced by order size. These include price discounts and transportation charges. Unfortunately, price discounts are usually not quoted as a continuous function of order size but as a step function, and therefore the calculus cannot be utilized to develop a simple decision rule. The common procedure used is one of comparing the total annual costs resulting from selected values for the order quantity, as it is shown by Hadley and Within [9]. But when quantity discounts apply for a family of items, the aggregate order quantity should be increased proportionately over the individual items and a maximum quantity that can economically be ordered to qualify for a discount is needed.

Another relevant cost is the carrying charge Meir [12] gives a detailed treatment of the effects of the holding rates and proposes that in most cases the practical expedient would be to assume the same holding rate for all items. Westing and Fine [22] explain the typical composition of the holding rate.

Since management's interest is more centrally focused on aggregate inventories and their behavior, it is very important that inventory control methods must be able not only to deal with policies and procedures to control individual item inventories but with a multiple item environment. This is the main reason why the research was focused to an area where practical applications may be considered.

CHAPTER II

MATHEMATICAL MODEL

Factors Considered

A model is presented for the situation in which replenishment stock is to be procured by purchasing. The source of replenishment stock is predetermined and is therefore not a policy variable. The demand rate and the procurement lead time are deterministic and time invariant. Demand and lead time are assumed to be independent of each other and independent of the procurement level and the procurement quantity.

The case to be studied is the grouping of orders for a number of individual items from one supplier. This is the case of joint ordering which has the effect of a common procurement cost of preparing the order and marginal costs associated with each item on the purchase order. A least cost policy for this system requires that all procurement cycles for items in a common procurement set must be of equal length. The initial problem is to identify all members of one family so that their lots can be delivered from the vendor at one time. The next step is to compute the average time between orders for this family, if the family is to be ordered in at least a T-year supply, so that the stock will last until the family is to be ordered again. For some items it may be desirable to order a 2T-year supply in every other time cycle, or a 3T-year supply in every third cycle. In general, the ith item will be ordered once every k_i cycles in a k_i T-year supply.

Relevant Cost Parameters

The ability to quantify and develop rigorous models of managerial problems is dependent on the determination of the behavior of relevant costs. The practical application of such models is also dependent on the ability to obtain the cost data that are needed. The result is that in many instances the relevant cost behavior for model-building purposes must be determined by special studies. The costs required to execute this joint order cost model are: Item Cost, Procurement Cost, and Inventory Holding Cost.

Item cost C_i, depends upon both the item and the source. In addition to the purchase, C_i also includes the cost of transportation, handling, service, delivery, and other charges that are assessed directly against the purchaser. In order to know that the price is properly determined, it is necessary to be familiar with the prices being asked for the item in question by acceptable suppliers, and it is also necessary to know the precise quality, or the quality range, that can be used by the buyer. Only if price quotations are known, including all related cost elements, and JF Quotations, relate the true cost to the quality requirements, is it possible to determine the true item prices.

Procurement cost C_p, includes the expenses of paper work preparation, communication, receiving, and vendor payment. This parameter depends upon the item as well as the source. When ordering an entire group of parts and materials from the same vendor, the cost C_p can be divided up into two components, a major cost, C_o, of preparing the order, paying the invoice, and such other costs that refer to the entire order, and minor costs, a_i, that are the marginal costs associated with each additional line item on the purchase order. Care must be taken, however, to be sure to obtain a true incremental cost of order preparation.

Inventory holding cost C_h, is incurred as a function of the number of units on hand and the time duration involved, it is an item-dependent parameter. There are handling costs required to place materials in inventory and to issue them from inventory and costs associated with storage, such as insurance, taxes, space, obsolescence, capital costs, deterioration, and losses. If average inventories increase, these costs will also increase and vice versa. All inventory is not subject to the same risk, obsolescence and deterioration. Therefore, the holding cost varies from item to item. This variation is expressed by means of a carrying charge r_i, which is formulated as a percentage per year of the item cost

$$r_i = H_i C_i \quad (\$/\text{unit})/\text{year}$$

where H_i is a holding rate determined by the associated holding costs. In most practical situations the holding rate is a constant value for each item in the inventory, $H_i = H$. Where very high cost items are involved the capital cost associated with the holding rate increases, and when holding costs which vary with the individual item are high, the value of H_i related to the cost of receiving, inspection, and handling can be identified by categorizing the parts into weight and size classes.

Mathematical Derivation

The model of these operations can be built up from the following arguments. The objective is to order a family of items, that come from the same vendor, once every T years that includes item i once every k_i cycles. The expected annual procurement cost including the major order cost C_o, and the minor marginal costs a_i, associated with each additional line of item i is

$$C_p = C_o \frac{1}{T} + \sum_{i=1}^{n} a_i \frac{1}{k_i T}$$

or

$$C_p = \frac{1}{T} \left(C_o + \sum_{i=1}^{n} \frac{a_i}{k_i T} \right) \quad (\$/\text{year}) \tag{1}$$

where the summation runs over all items n in the family.

The average lot ordered for item i every cycle time $k_i T$ is

$$q_i = k_i T R_i \quad (\text{units}) \tag{2}$$

The entire lot q_i, is delivered into stock at one time every cycle time $k_i T$.

From Figure 1, area A is the total number of unit-years of stock on hand during the inventory cycle T. If this area is divided by the number of years in a cycle to obtain the average on hand inventory during a year

$$\frac{k_i T R_i}{2} \quad (\text{units})$$

The holding cost per year is the assigned carrying charge r_i,

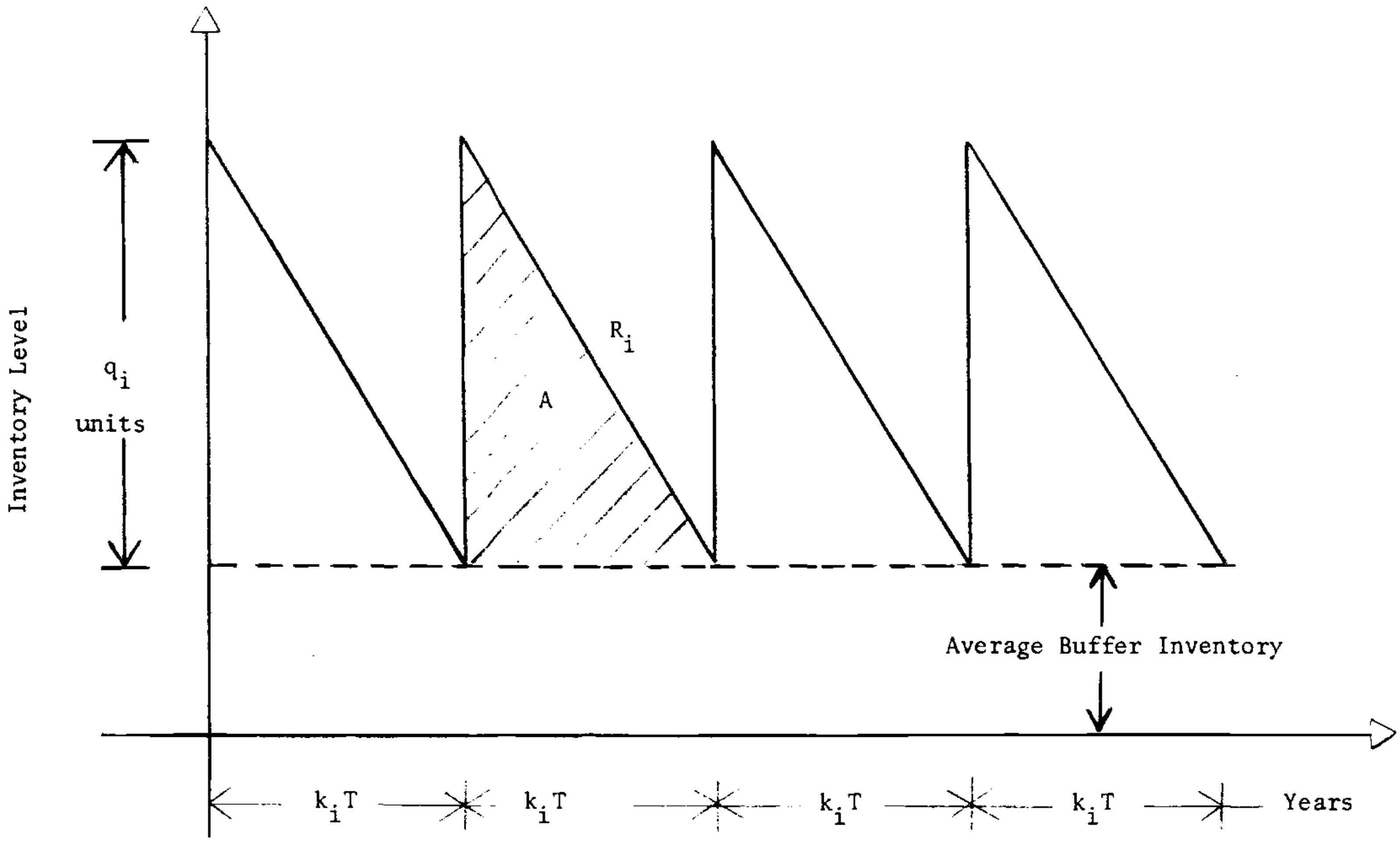


Figure 1. Inventory Process for Item i

which is the cost per unit held per year, times the annual average inventory on hand

$$C_h = \sum_{i=1}^{n} \frac{r_i k_i T R_i}{2} \quad (\$/\text{year}) \tag{3}$$

The total system cost per year will be the sum of the procurement cost for the year and the holding cost for the year symbolically represented as C_p and C_h and these costs are expressed as a function of the review period T and the cycle multiples k_i, therefore, the total expected annual cost is

$$TC = \frac{1}{T}\left(C_o + \sum_{i=1}^{n} \frac{a_i}{k_i}\right) + \frac{1}{2}\sum_{i=1}^{n} r_i k_i T R_i \left(\frac{\$}{\text{year}}\right) \tag{4}$$

The total cost is a function of two independent variables, T and k_i. Assume for the moment that the cycle multiples k_i are known.

The minimum total cost, with respect to the cycle time T, is obtained by taking the partial derivative of TC with respect to T and setting it equal to zero

$$\frac{\partial TC}{\partial T} = -\frac{1}{T^2}\left(C_o + \sum_{i=1}^{n} \frac{a_i}{k_i}\right) + \frac{1}{2}\sum_{i=1}^{n} r_i k_i R_i = 0$$

or

$$T = \sqrt{\frac{2\left(C_o + \sum_{i=1}^{n} \frac{a_i}{k_i}\right)}{\sum_{i=1}^{n} r_i k_i R_i}} \tag{5}$$

The cycle time thus depends on the cycle multiples k_i. Now suppose that the cycle time T is known, then the minimum total annual cost, with respect to the cycle multiple k_i, is obtained by taking the partial derivative of TC with respect to k_i and setting it equal to zero. The differentiation under the right-hand side summations of the total cost equation can be performed prior to the steps of summation from 1 to n

$$\frac{\partial TC}{\partial k_i} = - \frac{a_i}{k_i{}^2 T} + \frac{1}{2} r_i T R_i = 0$$

or

$$k_i = \frac{1}{T} \sqrt{\frac{2a_i}{r_i R_i}} \tag{6}$$

The cycle multiples k_i, that result will not usually be integers, but to preserve the order cycle, T, the values for k_i will be rounded to integers. A range of values for rounding the cycle multiples k_i to integers, can be derived as follows. The integer that minimizes total annual costs for item i, lies between two numbers, X and X + 1, for which costs are equal. These costs can be written as

$$TC_i(X) = \frac{a_i}{XT} + \frac{r_i X T R_i}{2}$$

and

$$TC_i(X+1) = \frac{a_i}{(X+1)T} + \frac{r_i (X+1) T R_i}{2}$$

where $X < k_i < X + 1$.

The condition is that $TC_i(X) = TC_i(X+1)$ or

$$\frac{a_i}{XT} + \frac{r_i XTR_i}{2} = \frac{a_i}{(X+1)T} + \frac{r_i(X+1)TR_i}{2}$$

which yields

$$X^2 + X - \frac{2a_i}{T^2 r_i R_i} = 0 \qquad (7)$$

Recalling that the value of k_i that minimizes total annual cost for item i is

$$k_i = \frac{1}{T}\sqrt{\frac{2a_i}{r_i R_i}}$$

Expression (7) becomes

$$X^2 + X - k_i^2 = 0 \qquad (8)$$

If the unit interval between X and X + 1 is to contain the integer 2, for example, the lower limit X must be at most 2, and the upper limit X + 1 at least 2. Solving for k_i at these limiting values for X we can obtain the range of values of k_i for which rounding should be to the integer 2.

$$k_i^2 = X^2 + X \qquad\qquad k_i = \sqrt{X^2 + X}$$

For the upper limit X + 1 = 2 or X = 1

$$k_i = \sqrt{1^2 + 1} = \sqrt{2} = 1.414$$

and the lower limit X = 2

$$k_i = \sqrt{2^2 + 2} = \sqrt{6} = 2.449$$

Thus, if k_i computed falls within the range 1.414 to 2.449 the value should be rounded to 2.

Therefore, the integer values to which the cycle multiples must be rounded can be found as shown, and a table can be constructed in order to know the range of the cycle multiples and the corresponding rounding convention to obtain integer values. Table 2 shows the results of these calculations for values of X from 2 to 8.

There is a slightly greater chance of rounding up to the next whole number than of rounding down to the next lower one. The range of values of k_i becomes narrower as the limiting values of (X) and (X + 1) increase. When the value of the tentative multiple k_i is larger than six, it will be acceptable to round-off using the integral part of k_i + 0.52 (see Table 3).

Cycle Times and Cycle Multiples

The iterative procedure for determining cycle times and cycle multiples is performed in the following steps:

1. Compute the initial estimate of the family cycle time (in the first iteration, assume all k_i = 1)

$$T = \sqrt{\frac{2\left(C_o + \sum_{i=1}^{n} \frac{a_i}{k_i}\right)}{\sum_{i=1}^{n} r_i k_i R_i}}$$

Table 2. Calculations of the Cycle Multiples Range of Values for Integer Rounding

Upper Limit (X+1) at Least and Lower Limit (X) at Most	Value of X	Range of Values of k_i $k_i = \sqrt{X^2 + X}$
2	1	1.414
	2	2.449
3	2	2.449
	3	3.469
4	3	3.469
	4	4.472
5	4	4.472
	5	5.477
6	5	5.477
	6	6.480
7	6	6.480
	7	7.483
8	7	7.483
	8	8.485

Table 3. Round-Off of the Cycle Multiples to Integer Values

Range of Values of Tentative Cycle Multiples $k_i = \frac{1}{T}\sqrt{\frac{2a_i}{r_i R_i}}$	Round to Integer Values
0 to 1.414	1
1.414 to 2.449	2
2.449 to 3.464	3
3.464 to 4.472	4
4.472 to 5.477	5
5.477 to 6.480	6

2. Use this value for T to obtain values for all k_i

$$k_i = \frac{1}{T}\sqrt{\frac{2a_i}{r_i R_i}}$$

3. The tentative multiple k_i will not usually be an integer. Round the value of each k_i to the appropriate integer using Table 3. If the rounded values for k_i are not identical to the values assumed in step 1, the steps are repeated. Recompute T using the new values for the cycle multiples k_i. Round these to the appropriate integer using Table 2 and check again for convergence. If none of the cycle multiples change the computations are finished. If some of the cycle multiples do change, then repeat step 1.

In the cases when equations (5) and (6) have a solution at all, the above iterative procedure will eventually converge to a minimum cost solution. The intuitive reason for this is that for any case under study, the value of C_o, a_i, r_i, and R_i will remain fixed, and the suggested algorithm starts to compute T assuming that all $k_i = 1$. Then using the value of T, the individual values of the k_i are computed. The new k_i are then used to correct the value of T. The effect of increasing some of the k_i will be to reduce T because the sum of the k_i will reduce the numerator and increase the denominator of the fraction that determines T. When the new value of T has been computed, new values of k_i are calculated. The effect of the new (smaller) value of T will be to increase the value of the k_i. As the cycle time is shortened the expression for the cycle multiples will increase, making the cycle time even shorter, until the computational cycle reaches a point where further reduction in the value of T does not change the values of the k_i. At this point it may be said that the values of T and k_i have converged to a solution.

The solution converges rapidly and only a few iterations are required for most problems. A flow chart describing the procedure to determine the cycle time T and cycle multiples k_i is presented in Figure 2.

Safety Stock Formulation

Even though the acquisition lead times and demand rates of all the items per family are known, it is wise to consider buffer stocks. The acquisition lead times might vary according to the supplier's conditions, and they are almost never known with absolute certainty, because this variable is source dependent. Thus, a buffer or safety

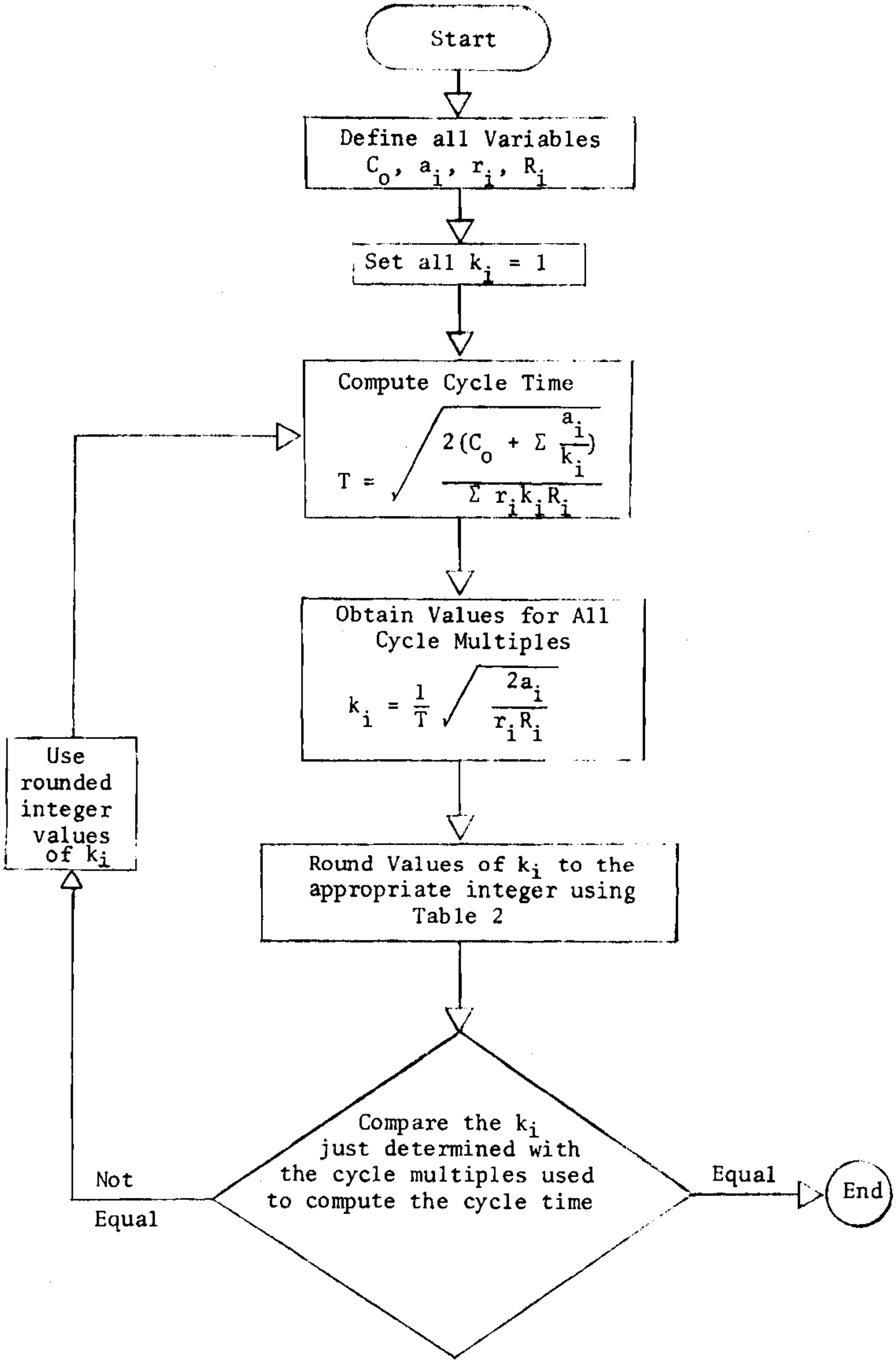


Figure 2. Flow Chart for Computing the Cycle Time and Cycle Multiples

stock is needed to absorb the variations in supply lead time.

The safety stock is simply the difference between the number of units used at maximum lead time τ_{max}, and average lead time $\tilde{\tau}$, with a constant demand D

$$SS = D\tau_{max} - D\tilde{\tau} \tag{9}$$

The safety stock will depend on the definition of lead time demand.

Lead time demand is the total demand which occurs during the lead time. When lead time is a random variable τ_x, lead time demand is a random variable Z_x. Safety stack computations are considerably simplified by assuming that the lead time demand distribution follows some definite mathematical function, $f(Z_x)$. In general, both demand and lead time will take on integral values. This leads to the conclusion that lead time demand will also be an integer-valued random variable process with independent increments, usually called a generalized Poisson process

$$f(Z_x) = f(D_x, \tau_x)$$

or

$$Z_x = D_x \tau_x$$

It can be seen that $f(Z_x)$ is a function of two random variables and is thus a result of a joint distribution. If the distributions of D_x and Z_x are given, we can develop the distribution of Z_x by convulution.

For any but the simplest density functions, this procedure takes a considerable computational effort. If demand is constant over time, it is possible to obtain the distribution of lead time demand $f(Z_x)$, by direct mathematical means. Selected convulutions for compatable density functions are given in Table 4.

The normal, Poisson, and nagative exponential distributions have been found to be of considerable value in representing demand functions for inventory management. The normal distribution has been found to describe many demand functions adequately, particularly at the factory level of the supply-production-distribution system. This assumption may not be true for some of the items, but, essentially, it will take care of the majority of the cases.

The determination of the safety stock using the lead time demand distribution is as follows

$$SS = Z_{max} - \tilde{Z} \tag{10}$$

where Z_{max} is the maximum lead time demand and $\tilde{Z}$ is the mean lead time demand.

Given the assumption of normality Z_{max} can be determined by reference to the normal distribution tables where $Z_{max} = \tilde{Z} + F\sigma_z$ and F measures the departure of Z from the mean $\tilde{Z}$, in units of standard deviations, σ_z. One must know only mean lead time demand $\tilde{Z}$ and the standard deviation σ_z to completely describe a normal distribution demand. Table 5 shows some of the probabilities that the maximum lead time demand Z_{max} will exceed $\tilde{Z} + F\sigma_z$ for selected values of F.

Table 4. Lead Time Demand Cases for Constant Demand and Probabilistic Lead Time

Lead Time Distribution τ_x	Parameters	Lead Time Demand Distribution Z_x	Parameters
Normal	μ, σ^2	Normal	$D\mu,\ D^2\sigma^2$
Poisson	μ	Poisson	$D\mu$
Exponential	β	Exponential	$D\beta$
Gamma	α, β	Gamma	$\alpha,\ D\beta$
Chi-square	η	Chi-square	$\frac{\eta}{2} - 1,\ 2D$

Table 5. Probability that Lead Time Demand Exceeds Z_{max} for Selected Values of F

Z_{max} $\tilde{Z} + F\sigma_z$	Probability
$\tilde{Z} + 3.090\ \sigma_z$	0.001
$\tilde{Z} + 2.576\ \sigma_z$	0.005
$\tilde{Z} + 2.326\ \sigma_z$	0.010
$\tilde{Z} + 1.960\ \sigma_z$	0.025
$\tilde{Z} + 1.645\ \sigma_z$	0.050
$\tilde{Z} + 1.232\ \sigma_z$	0.100
$\tilde{Z} + 1.036\ \sigma_z$	0.150
$\tilde{Z} + 0.842\ \sigma_z$	0.200
$\tilde{Z} + 0.674\ \sigma_z$	0.250
$\tilde{Z} + 0.524\ \sigma_z$	0.300
$\tilde{Z} + 0.385\ \sigma_z$	0.350
$\tilde{Z} + 0.235\ \sigma_z$	0.400
$\tilde{Z} + 0.126\ \sigma_z$	0.450
$\tilde{Z}$	0.500

From Equation (10) and the definition of Z_{max}

$$SS = F\sigma_z \tag{11}$$

In cases where Z has a normal or Poisson lead time demand distribution the computation of σ_z involves the square root of $\tilde{Z}$, the mean lead time demand. In this event safety stocks might vary as the square root of lead time demand. Thus, if $\tilde{Z}$ increases or decreases reserve stocks should not change in direct proportion, but in the proportion to the square root. Thus, there is an economy of scale involved and a larger demand can be accommodated by a less than proportional safety stock.

The larger the safety stock the smaller will be the risk of running out of stock, but the problem is to determine a method which will allow to set safety stocks at reasonable levels such that the risk of stock-out is acceptable.

Though it is not difficult to develop a model for safety stock based on the concept of balancing inventory and stockout cost, more often than not it is difficult or impossible for management to isolate a realistic stockout cost. Stockouts are highly undesirable but there is no reasonable basis for considering stockout cost in the calculation of a safety stock. What is needed is another way to determine the safety stock level.

One approach is to classify all items into several categories depending upon the seriousness of their shortage. Items in category I will have a higher safety stock level than items in category II. The amount of safety stock needed for an item will depend on the number of stockouts per year that will be tolerated and on the number of times the item is ordered during the course of a year, $(1/k_iT)$. Table 6 shows the probability of a stockout at each ordering instance as a function of the number of deliveries per year and the permissible number of stockouts. Any number of deliveries less than the permissible number of stockouts, gives us no risk or zero probability of exceeding the acceptable number of stockouts. Using this approach management can state the chances that a stockout will occur either for a single item i or a lot q_i. In this particular case, the number of stockouts per year that can be afforded for a lot q_i has been calculated, and the number of deliveries per year is known from the cycle time T. Thus, the lots q_i can be classified according to their needs and values in the inventory stock.

For example, consider a lot for which only two stockouts per year can be tolerated. If there are eight deliveries in a year for this lot, it will be susceptible to a stockout eight times in one year. Since it is acceptable to tolerate only two stockouts in one year, the odds of having a stockout are 8:2 or 4:1, in other words there is one failure in five chances and the probability that a stockout will occur should be 1/5 or 0.20. Thus, there is an acceptable 20% risk of running out of stock. From Table 5 the maximum lead time demand Z_{max}, exceeds $\tilde{Z} + F\sigma_z$ by 0.2 or 20% when F = 0.842.

To implement a category safety stock for any item, it is necessary to determine the corresponding value of F for the stockout probability of the selected category and proceed to compute the safety stock by means of Equation (11), where F is called the stockout acceptance factor and it is based upon the lead time demand distribution. In the

Table 6. Probability of Stockouts

Category	I	II	III	IV
Permissible Stockouts per year	1	2	3	4
Deliveries per year	Probability of Stockouts			
1	0.5000			
2	0.3333	0.5000		
3	0.2500	0.4000	0.5000	
4	0.2000	0.3333	0.4285	0.5000
5	0.1666	0.2857	0.3757	0.4444
6	0.1428	0.2500	0.3333	0.4000
7	0.1250	0.2222	0.3000	0.3636
8	0.1111	0.2000	0.2727	0.3333
9	0.1000	0.1818	0.2500	0.3076
10	0.0909	0.1666	0.2307	0.2857
11	0.0833	0.1538	0.2142	0.2666
12	0.0769	0.1428	0.2000	0.2500

case of a normal lead time demand distribution, F can be determined as shown in Figure 3, where the acceptable stockout percentage is the stockout probability of Table 5 which is used in conjunction with the Cumulative Normal Distribution Tables to give the corresponding values of F.

The safety stock corresponding to a family of items is computed as the summation of every item safety stock for all the items in the family

$$SS_F = \sum_{i=1}^{n} SS_i \tag{12}$$

where SS_i is the safety stock of item i determined by Equation (11).

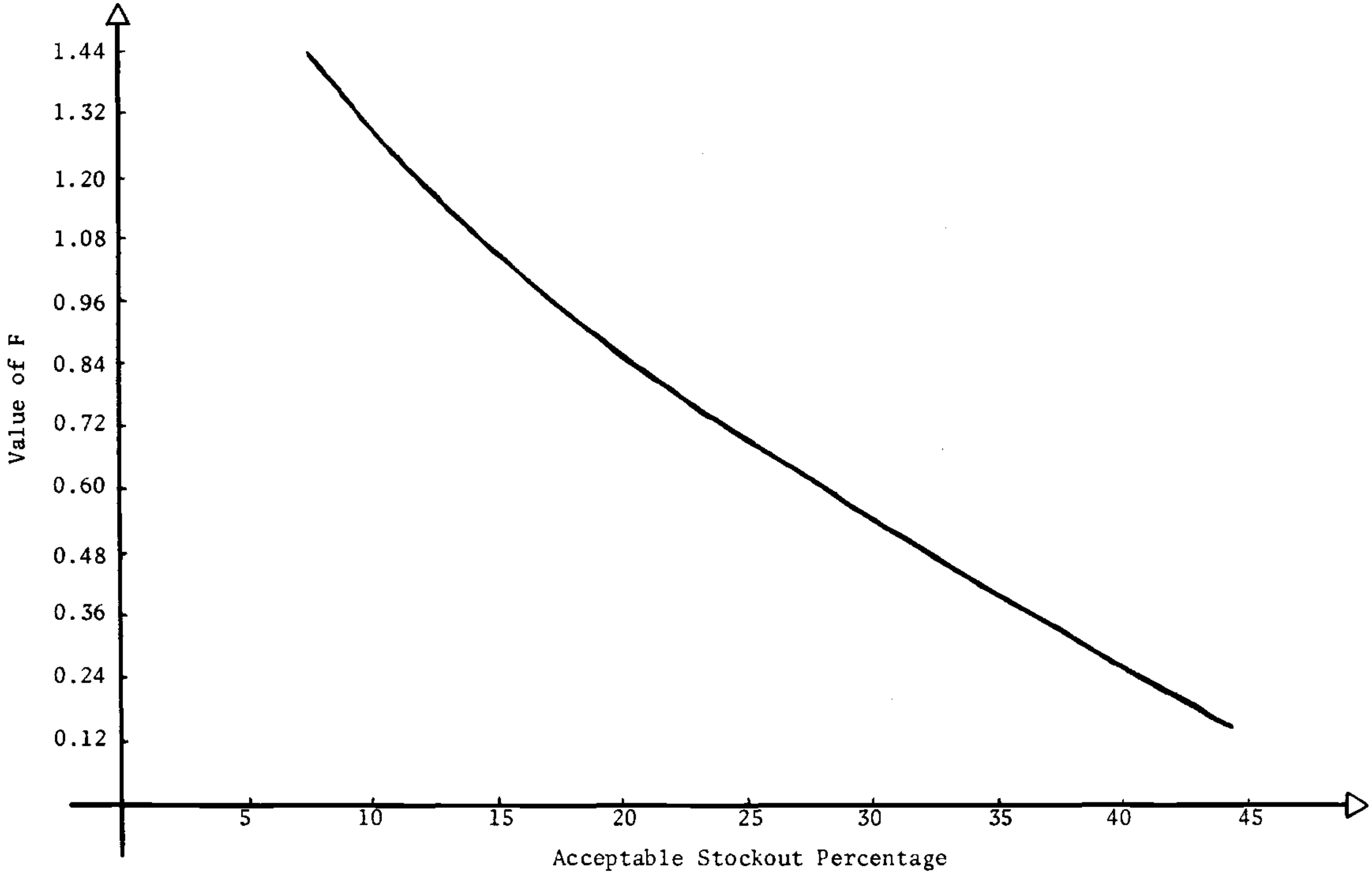


Figure 3. Stockout Acceptance Factor F

CHAPTER III

SENSITIVITY ANALYSIS AND OTHER CONSIDERATIONS

Sensitivity of Cost Parameters

Once the mathematical model is built it can be used as a decision model, but before any decision is made it is important to explore the effects of estimating inaccuracies. The purpose of the sensitivity analysis on the cost parameters is to find out how badly the system would be penalized by estimating errors. Sensitivity analysis is useful to assess the impact of inexact information and to guide the allocation of limited resources.

Errors may result in estimating parameter values, that cause the calculated values of the policy variables to be nonoptimum. A measure of the effect of inaccurately estimating parameter values is the percentage deviation in cost caused by a deviation in the estimated values of the ordering cost C_o, marginal cost a_i, and carrying charge r_i. Symbolically,

$$B = \frac{TC' - TC}{TC} \times 100$$

where

B = the measure of sensitivity

TC' = the true total yearly cost

TC = the minimum total yearly cost

To determine the effect of inaccurately estimating a particular parameter, all other parameters should be fixed, the parameter in question changed a known amount, and the effect on B noted.

Ordering cost C_o is the first cost parameter to be examined. If all other parameters are held fixed, the values of the policy variables based on the parameter estimates may be determined as

$$T' = \sqrt{\frac{2\left(C_o' + \Sigma \frac{a_i}{k_i}\right)}{\Sigma r_i k_i R_i}}$$

and

$$k_i' = \frac{1}{T} \sqrt{\frac{2a_i}{r_i R_i}}$$

substituting these values into the total cost equation (4), yields

$$TC' = \frac{1}{T'} (C_o + \Sigma \frac{a_i}{k_i'}) + \frac{1}{2} \Sigma r_i k_i' T' R_i$$

We now can determine the percentage deviation in cost caused by a deviation in the estimated value of C_o'. The measure of sensitivity for the cost parameter C_o is

$$B = \frac{TC' - TC}{TC} \times 100$$

A sensitivity curve for B is exhibited in Figure 4, where the estimated value C_o' was varied with a range of 0.5 to 2.0 of the true values.

Marginal cost a_i is the second cost parameter considered. Fixing the values for the other parameters and changing marginal cost affects the values of

$$T' = \sqrt{\frac{2\left\{C_o + \sum_{i=1}^{n} \frac{a_i'}{k_i}\right\}}{\sum_{i=1}^{n} r_i k_i R_i}}$$

and

$$k_i' = \frac{1}{T'} \sqrt{\frac{2a_i'}{r_i R_i}}$$

substituting these values into the total cost equation (4), yields

$$TC' = \frac{1}{T'}\left\{C_o + \sum_{i=1}^{n} \frac{a_i}{k_i'}\right\} + \frac{1}{2}\sum_{i=1}^{n} r_i k_i' T' R_i$$

The measure of sensitivity for the cost parameter a_i is

$$B = \frac{TC' - TC}{TC} \times 100$$

A sensitivity curve for B is exhibited in Figure 5, where the estimated value a_i' was varied with a range of 0.5 to 2.0 of the true values.

Carrying charge r_i is the final cost parameter considered. Changing the values of the holding rate I affects the policy variables, the result being

$$T' = \sqrt{\frac{2\left(C_o + \sum_{i=1}^{n} \frac{a_i}{k_i}\right)}{\Sigma\, r_i' k_i R_i}}$$

and

$$k_i' = \frac{1}{T'}\sqrt{\frac{2a_i}{r_i' R_i}}$$

substituting these values into the total cost equation (4), yields

$$TC' = \frac{1}{T'}\left(C_o + \sum_{i=1}^{n} \frac{a_i}{k_i}\right) + \frac{1}{2}\sum_{i=1}^{n} r_i k_i' T' R_i$$

The measure of sensitivity for the cost parameter r_i is

$$B = \frac{TC' - TC}{TC} \times 100$$

A sensitivity curve for B is exhibited in Figure 6, where the estimated value r_i' was varied with a range of 0.5 to 2.0 of the true values.

Table 7 shows the results of the sensitivity analysis performed on the three parameters of the mathematical model. All the numerical values were calculated by the use of a computer program, listed in Appendix A, on the Univac 1108 at the Computer Center of Georgia Institute of Technology.

Graphs of how the calculated value of total inventory cost is affected by error in the estimation of parameters, are shown in Figures 4, 5, and 6. These figures were based on the numerical values obtained

Table 7. Cost Deviations in Sensitivity Analysis

Range of Errors	Cost Deviations (%) for C_o'/C_o	a_i'/a_i	r_i'/r_i
0.50	2.88	1.28	6.07
0.60	1.70	0.34	3.28
0.70	0.84	0.14	1.59
0.80	0.37	0.08	0.62
0.90	0.13	0.05	0.14
1.00	0.00	0.00	0.00
1.10	0.07	0.06	0.11
1.20	0.26	0.08	0.42
1.30	0.51	0.12	0.86
1.40	0.83	0.17	1.42
1.50	1.20	0.28	2.06
1.60	1.72	0.39	2.77
1.70	2.18	0.48	3.54
1.80	2.97	0.60	4.35
1.90	3.78	0.71	5.19
2.00	4.30	0.82	6.07

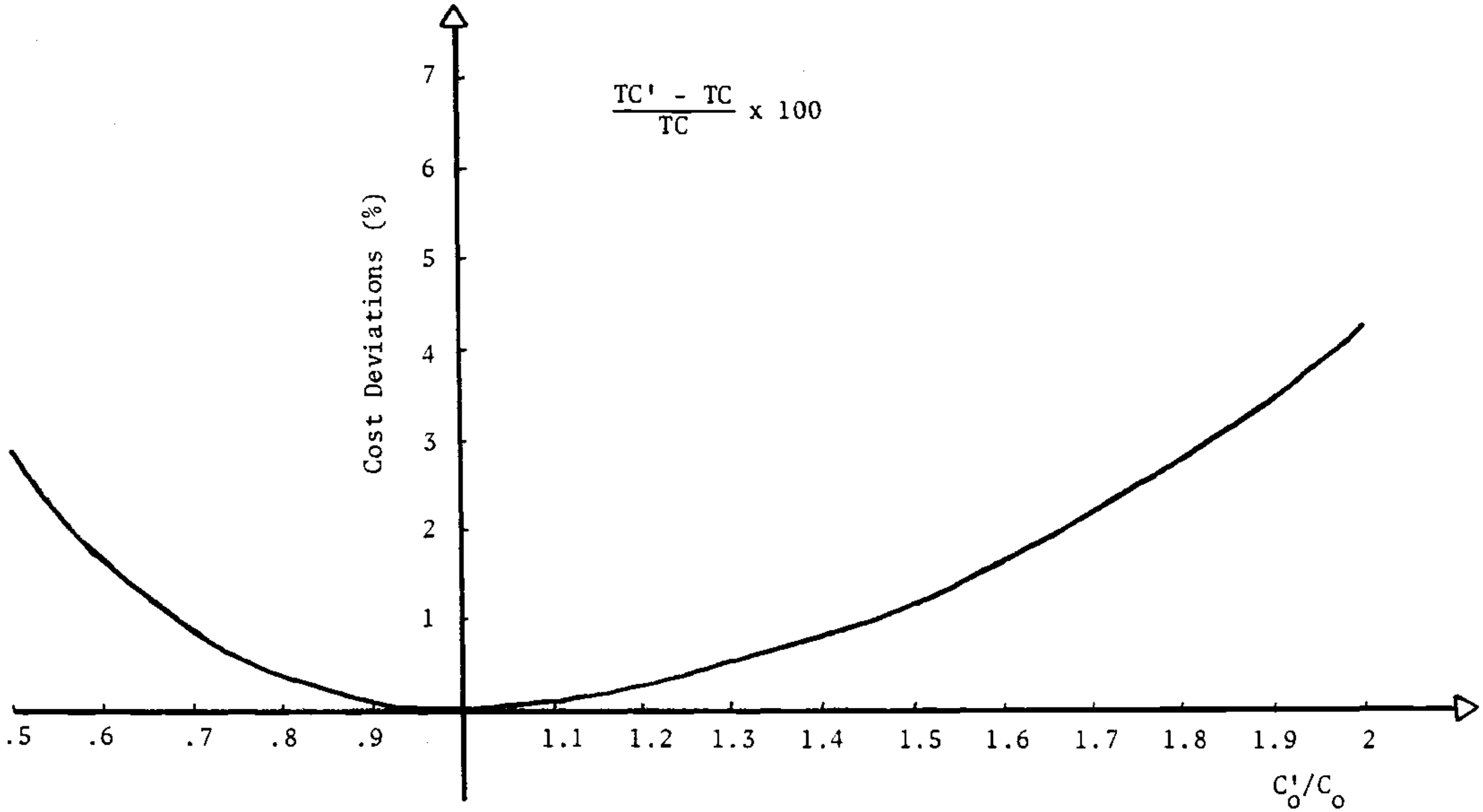


Figure 4. Total Inventory Cost Affected by Errors in the Estimation of Ordering Cost C_o

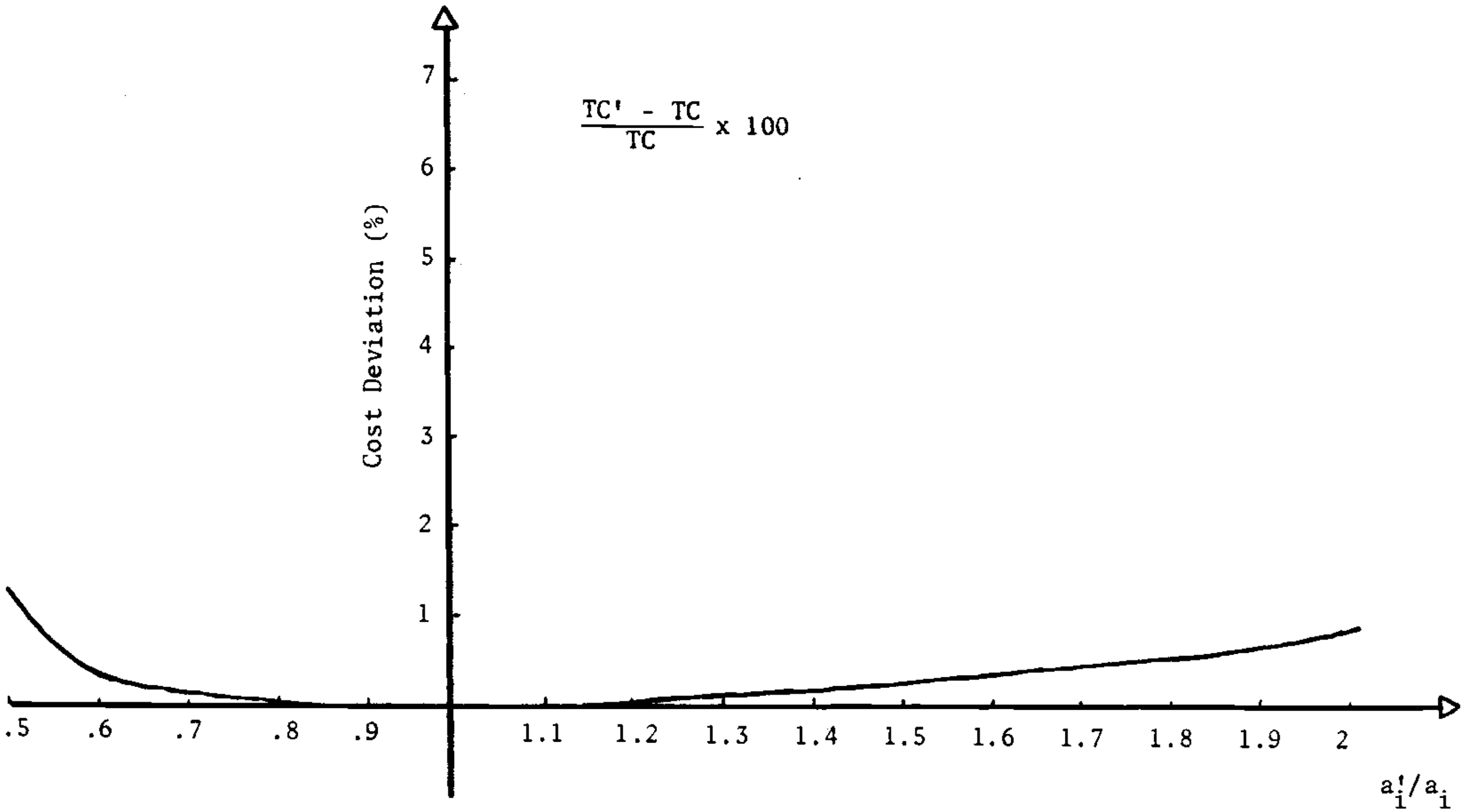


Figure 5. Total Inventory Cost Affected by Errors in the Estimation of Marginal Cost a_i

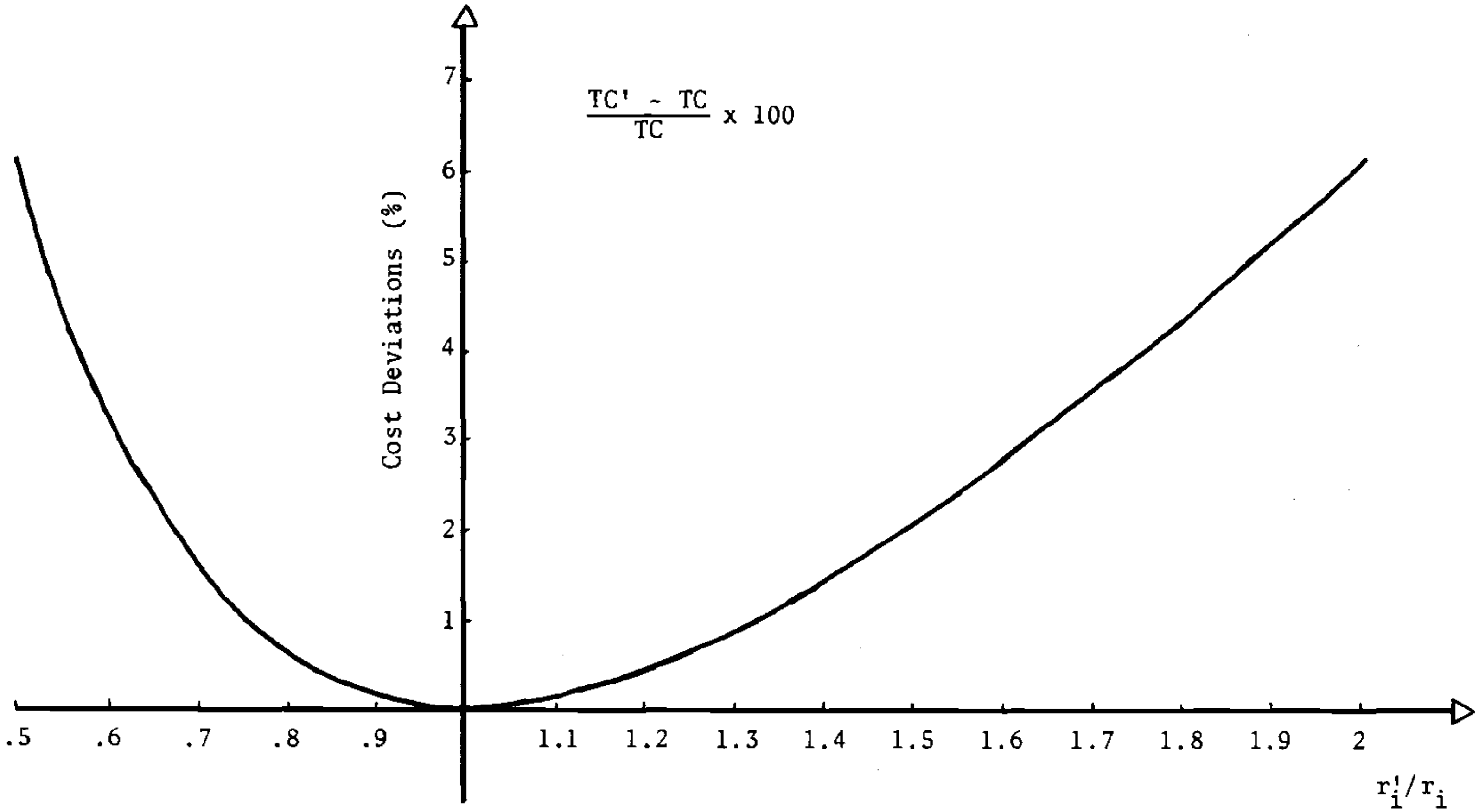


Figure 6. Total Inventory Cost Affected by Errors in the Estimation of Carrying Charge r_i

from Table 7, and were used as a reference in the selection of exact cost parameters.

The sensitivity analysis gave an idea of the amount of effort that should be spent on each parameter's estimation. Figures 4, 5, and 6 show that in general underestimation is more dangerous than overestimation. An error in marginal cost a_i does not have much effect on total cost. In the case of ordering cost and carrying charge estimation, the total cost is rather sensitive (somewhat more for underestimation than overestimation), so more time should be spent estimating C_o and r_i accurately.

Cycle Multiples as a Function of the Carrying Charge

In the case where the carrying charge is the same for all items, and if management chooses to increase the carrying charge, it is easy to see from the equation for the cycle time T that the larger the carrying charge r_i, the shorter the cycles, these formulas will generate more orders per year, with smaller working stocks on the average. But the cycle multiples k_i are independent of these changes; they will remain the same. This is demonstrated in the following manner.

Substituting the expression for T into the equation for computing the cycle multiples k_i we obtain

$$k_i = \frac{1}{T}\sqrt{\frac{2a_i}{r_iR_i}} = \sqrt{\frac{\Sigma\ r_ik_iR_i}{2\left(C_o + \Sigma\frac{a_i}{k_i}\right)}}\ \sqrt{\frac{2a_i}{r_iR_i}}$$

where $r_i = HC_i$ and the holding rate H is the same for all items; therefore,

$$k_i = \sqrt{\frac{H \, \Sigma \, C_i k_i R_i}{2\left(C_o + \Sigma \dfrac{a_i}{k_i}\right)} \frac{2a_i}{H \, C_i R_i}} = \sqrt{\frac{a_i \, \Sigma \, k_i R_i C_i}{\left(C_o + \Sigma \dfrac{a_i}{k_i}\right) R_i C_i}}$$

The holding rate H cancels out and does not appear in the formula. The multiples do not change when the family cycle time is changed. Each item in the family is ordered in a smaller quantity, but in the same proportion to the reduced cycle.

Quantity Discounts in Joint Ordering

Once the family of items to come from a vendor has been selected, the cycle time and the cycle multiples for each item in the joint group can be computed together with the average lot ordered for item i each cycle.

There are cases where vendors offer a quantity discount, not on single items, but on the total order value or on the quantity of all items ordered for delivery at one time. An increase in the order quantity q_i, increases the time supply for item i but does not change its cycle multiple k_i. Thus, if the aggregate order quantity should be increased, the increase is spread proportionately over the individual items in the family. Therefore, an increase in total order quantity, which increases the time supply for the family, would be multiplied by the same k_i to get individual item quantities.

To derive a logic for deciding whether or not to increase the total order quantity to qualify for a discount, first express the total order quantity in some common unit of measure. If the discount is basically a freight savings, the order quantities can be expressed in weight units. If the discount is based on ordering a minimum value, express the order quantities in dollars. But if there are some cycle multiples k_i different from the unit value, the total order quantity will not be the same for every cycle time T.

Computing the total order quantity for every cycle is a long and tedious procedure that gets more cumbersome as the number of cycles per year increases. It is particularly complicated for large families of items. It is assumed that because cycle multiples other than $k_i = 1$ occur only for very low dollar items, the simple sum of the usage rates in dollar value $\Sigma R_i C_i$ and the weighted sum $\Sigma k_i R_i C_i$ have a difference which is generally negligible. Thus, for purposes of quantity discounts the cycle multiples k_i can be assumed equal to 1 and the total order quantity per cycle can be determined by (1) adding up all the individual annual usages either in value or units, for every item in the family; and (2) multiplying this total sum by the cycle time in years. This gives the expression for the total order quantity per cycle

$$QD = \sum_{i=1}^{n} R_i C_i T \quad \text{(dollars)} \tag{13}$$

or

$$Q = \sum_{i=1}^{n} R_i T \quad \text{(units)} \tag{14}$$

Since the same number of items will be ordered per family, the marginal costs a_i are not considered in the ordering cost. Each time a quantity Q is ordered the cost is $C_o + QD$, and the present value of this ordering cost to be incurred Y years in the future is

$$P = (C_o + QD)\left[\frac{1}{e^{HY}}\right] = (C_o + QD)e^{-HY} \tag{15}$$

where H is the holding rate or the annual interest rate, Y is the cycle time in terms of the total order quantity, and total annual usage

$$Y = Q/S$$

where

$$S = \Sigma R_i$$

Assuming that the first order is placed now, and subsequent orders are placed as the stock is used up, the present value of all these expressions is

$$P(QD) = (C_o + QD)(1 + e^{-HY} + e^{-2HY} + e^{-3HY} + \ldots) \tag{16}$$

the second term of the right-hand side of this expression is similar to the series

$$1 + a^1 + a^2 + a^3 + \ldots = \frac{1}{1 - a}$$

where a = e and the exponent is -HY or

$$1 + e^{-HY} + e^{-2HY} + e^{-3HY} + \ldots = \frac{1}{1 - e^{-HY}}$$

Then, expression (15) becomes

$$P(QD) = \frac{(C_o + QD)}{1 - e^{-HY}}$$

Consider the case where the vendor offers a discount for buying in a larger quantity $Q_2 > Q_1$, and the effective unit cost is $C_{2i} < C_{1i}$. The present expected value of always ordering in the larger lot Q_2 is

$$P(QD_2) = \frac{C_o + QD_2}{1 - e^{-HY_2}}$$

where $QD_2 = \Sigma R_i C_{2i} T$ and $Y_2 = Q_2/S$.

The larger lot Q_2 is to be compared with the normal lot Q_1, which is the total order quantity per cycle just determined. The quantity Q_2 is worthwhile only if

$$\frac{C_o + QD_2}{1 - e^{-HY_2}} < \frac{C_o + QD_1}{1 - e^{-HY_1}} \tag{17}$$

where $QD_1 = \Sigma R_i C_{1i} T$ and $T_1 = Q_1/S$.

The cost of processing an order C_o is usually very much smaller than the total invoice price QD so it could be ignored in this comparison

$$\frac{QD_2}{1 - e^{-HY_2}} < \frac{QD_1}{1 - e^{-HY_1}} \tag{18}$$

From the power series

$$a^{-X} = 1 - X + \frac{X^2}{2!} - \frac{X^3}{3!} + \ldots$$

we can notice that if X = HY and a = e, the expression becomes

$$e^{-HY} = 1 - HY + \frac{(HY)^2}{2!} - \frac{(HY)^3}{3!} + \ldots$$

and neglecting terms from the cube onwards

$$1 - e^{-HY_1} = 1 - \left[1 - HY_1 + \frac{(HY_1)^2}{2!}\right] = HY_1 - \frac{(HY_1)^2}{2}$$

substituting the value of Y_1

$$1 - e^{-HY_1} = \frac{HQ_1}{S}\left(1 - \frac{HQ_1}{2S}\right)$$

similarly

$$1 - e^{-HY_2} = \frac{HQ_2}{S}\left(1 - \frac{HQ_2}{2S}\right)$$

Then expression (16) becomes

$$\frac{QD_2}{\frac{HQ_2}{S}\left(1 - \frac{HQ_2}{2S}\right)} < \frac{QD_1}{\frac{HQ_1}{S}\left(1 - \frac{HQ_1}{2S}\right)}$$

or

$$\frac{QD_2}{QD_1} < \frac{Q_2}{Q_1}\left(\frac{2S - HQ_1}{2S - HQ_2}\right) \qquad (19)$$

Expressing the unit prices in terms of a discount fraction

$$C_{2i} = (1 - d)C_{1i}$$

produces

$$QD_2 = \Sigma R_i C_{2i} T_2 = (1 - d) T_2 \Sigma R_i C_{1i}$$

and

$$QD_1 = T_1 \Sigma R_i C_{1i}$$

therefore

$$\frac{QD_2}{QD_1} = \frac{(1 - d) T_2 \Sigma R_i C_{1i}}{T_1 \Sigma R_i C_{1i}} \tag{20}$$

Since the summation $R_i C_{1i}$ appears in both the numerator and denominator, it can be cancelled. Knowing that

$$T_1 = Q_1/\Sigma R_i \quad \text{and} \quad T_2 = Q_2/\Sigma R_i$$

Equation (22) may be written as

$$\frac{(1 - d) Q_2}{Q_1} < \frac{Q_2}{Q_1} \left(\frac{2S - HQ_1}{2S - HQ_2} \right)$$

which can be reduced to

$$Q_2 < \frac{2dS}{H} + (1 - d) Q_1 \tag{21}$$

and changing the expression to an equality produces

$$Q_2 = \frac{2dS}{H} + (1 - d) Q_1 \tag{22}$$

where Q_2 is the maximum quantity that can economically be ordered to qualify for a discount.

This expression does not apply for absurdly large discounts, because it was assumed in the derivation that ordering cost C_o is small compared with the family dollar value QD, which would not be true if the item costs C_i were quite small.

Equation (22) can be used to determine whether it is worthwhile to apply for a discount. The quantity Q_1 is the original family total ordering quantity, and if the quantity being offered with a price discount of d percent is less than the value of Q_2, then one should order the quantity the vendor is offering. In fact, one could order up to Q_2 units and still realize a benefit from the price discount.

CHAPTER IV

EXERCISING THE MODEL WITH ACTUAL DATA

System Environment

Following is a brief description of the kind of environment where this joint ordering inventory model might be applied. The line of products that the company supplies its customers is standardized at least during a year or two. From one year to the next, some old products are dropped from the line and other new ones are added. The added products may be substitutes for, or improvements on the older products, or they may be entirely new items. There are a few of the company's customers which but the whole production and the demand is forecast by means of yearly contracts. The company may have to order raw materials and components in advance of its own needs because of the lead times required by its suppliers, and there are cases where a large number of items are regularly ordered from common suppliers. The manufacturing processes for the different items share common machinery and facilities. And the finished items may be sent from the plant to storage in a factory warehouse and subsequently shipped from the warehouse to customers. These characteristics of the environment have critical bearing on the requirements for an effective inventory control system.

Data Collection

To test the model, the following data were collected regarding a sample of 100 items distributed in ten families:

Part Number	Vendor
Annual Usage	Mean Lead Time
Unit Cost	Standard Deviation

All the collected information about these parts was used in testing the model, but extra data can be needed for designing other controls in the system. Table 8 shows the information regarding this sample of ten families.

Another important set of data needed at this point is cost of ordering, marginal costs, and the carrying charges.

Each time a purchase order is issued, the purchasing, inspection, inventory control, receiving, and accounts payable departments must service it. For determining the correct ordering policies one must calculate the extra cost of servicing this one order. It is this marginal cost rather than the average cost per order that is important because the fixed costs of these departments continue, regardless of the number of orders written. An ordering cost was determined which consisted of the cost of preparing a requisition and a purchase order and the cost of accounts payable expenses per purchase order plus the marginal costs associated with each additional line of item in the purchase order. Each of these costs is marginal or incremental; it is not based on total cost of running the Inventory and Procurement Department. It was estimated that the ordering cost C_o had a value of ten dollars per order, and the marginal costs a_i were forty cents per additional line of item.

The holding rate H of the carrying charge r_i was found to be associated with the following cost categories:

Table 8. Data Sample of Ten Families

Family Supplier	Normal Distributed Lead Time: Mean $\tilde{T}$	Normal Distributed Lead Time: Standard Deviation σ	Number of Items per Family N	Total Family Demand $D = \sum^{N} R_i$
1	3.00	1.75	9	63,200
2	4.00	2.00	13	5,780
3	4.00	2.25	10	52,325
4	2.00	1.41	8	17,870
5	3.50	1.66	13	955
6	2.75	1.87	7	21,355
7	4.50	2.02	8	2,170
8	3.75	1.80	12	32,905
9	2.80	1.76	6	7,215
10	4.00	2.28	14	27,815

Capital Cost. It is the annual interest rate foregone on the dollars tied up in inventory.

Insurance. The cost of insuring the inventory goods.

Tax. This is tax paid on the fixed assets of the crib and not the inventory size.

Handling. This was estimated to be 20% of the total labor costs of the receiving and stores department, which is equivalent to 5% of the average inventory value.

Space. The total space of the receiving and stores department is adjusted by subtracting the office space, the space allotted to receiving and the space used in the filling of orders. The remaining space is extended by the standard cost per square foot to arrive at the total yearly storage cost. This is represented as a percentage of average inventory.

Obsolescence. The estimated obsolescence expense for a year was 6% of the average inventory value. This figure was the greatest cost element due to the technological changes in the product design.

Deterioration and Losses. These are the costs associated with either the mysterious disappearance of items of inventory or their deterioration due to overextended shelf time.

The holding rate is the total of all the above and equals 24% of the item cost per year. It is surprisingly costly to carry inventory. The individual elements of costs are the following:

Cost Elements	Annual % of Item Cost
Capital Cost	5.5
Insurance	0.5
Tax	2
Handling	5
Space	4
Obsolescence	6
Deterioration and Losses	1
Holding Rate	H = 24.0

Evaluation of the Model

Using the above data the total cost of the model can be calculated and compared with the total cost involved with the procurement policies currently in use. The current procurement policies state the independent determination of the order quantities q_i, using the basic inventory formula for a fixed reorder quantity system, where the order quantity is determined by

$$q_i = \sqrt{\frac{2R_iC_o}{r_i}}$$

and the average annual cost of procurement and holding inventory for item i is

$$C(q_i) = \sqrt{2R_iC_or_i}$$

Another important comparison is the average inventory level held during a year. In the current procurement policies this is determined by the following formula

$$AI = q_i/2 + SS_i$$

where the safety stock is a month supply for item i.

But in order to calculate this average inventory for the new model one must know first the exact family total order quantity for every cycle time in any period of time.

Determining the Family Total Order Quantity per Cycle

The total quantity ordered every cycle time T can be determined in the following manner. First start with cycle number one where the total order will be the summation of all the lots q_i ordered in the family

$$TOQ_1 = \Sigma q_i$$

In subsequent cycles the total quantity ordered will vary according to the lots q_i ordered every k_i cycles in a $k_i T$ - year supply. For example in cycle time number six, the total order quantity is the summation of all the lots q_i ordered every cycle $k_1 = 1$ plus the lots ordered every other cycle $k_2 = 2$, together with the lots ordered every third cycle $k_3 = 3$ and every sixth cycle $k_6 = 6$. To calculate the total order quantity for cycle time number J, it begins in cycle number two and is as follows:

$$TOQ_j = \Sigma p_{ij} q_i \quad j = 2,3,4,5,\ldots,NC \qquad (23)$$

where NC is the number of cycles desired to compute, $p_{ij} = 1$ for all cycle multiples k_i equal to the corresponding values of the submultiples of cycle time number J and all other p_{ij} are set equal to zero. These coefficient values are calculated by the following formula

$$KJL = [J/L] \quad \text{[integer values only]}$$

where J is the cycle time number that begins in the second cycle and L = J,J-1,J-2,J-3,...,3,2,1. The limiting value of KJL is the greatest value KG of all the cycle multiples k_i in the family. Therefore, the submultiples interval for cycle time number J starts with the value one and ends with the value of KG. Table 9 shows the values of KJL for up to 20 cycle times.

An Example. Table 10 shows a family composed of 13 items with their annual requirements and the resulting cycle multiples k_i, cycle time T, and the lots q_i ordered for each item i every cycle time k_iT. The greatest cycle multiple is five, thus the limiting value of KJL is five and the submultiples interval is from one to five.

To compute the total quantity ordered in cycle four, from Equation (23),

$$TOQ_4 = \sum_{1}^{13} p_{i4}q_i$$

Using Table 9 in conjunction with Table 10 the values of p_{i4} can be determined for all k_i equal to the submultiples KJL

$$p_{14} = p_{24} = p_{34} = p_{44} = p_{54} = p_{64} = p_{74} = p_{11,4} = 1$$

where all the other $p_{i,4} = 0$, therefore

Table 9. Ordering Schedule for up to 20 Cycle Multiples
and 20 Cycle Times

J	Values of KJL for Cycle Time Number J																			
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	1	2																		
3	1		3																	
4	1	2		4																
5	1				5															
6	1	2	3			6														
7	1						7													
8	1	2		4				8												
9	1		3						9											
10	1	2			5					10										
11	1										11									
12	1	2	3	4		6						12								
13	1												13							
14	1	2					7							14						
15	1		3		5										15					
16	1	2		4				8								16				
17	1																17			
18	1	2	3			6			9									18		
19	1																		19	
20	1	2		4	5					10										20

Table 10. Family Example of 13 Items

Item Number	Unit Cost	I = 0.20 Carrying Charge	Annual Require-ments	T = 0.4694 Cycle Multiples	Lot Ordered
i	C_i	r_i	R_i	k_i	q_i
1	0.72	0.144	654	1	307
2	0.66	0.132	43	1	20
3	0.26	0.052	104	1	49
4	0.25	0.050	77	1	36
5	0.94	0.188	13	1	6
6	0.53	0.106	13	2	12
7	0.54	0.108	12	2	11
8	0.91	0.182	4	3	6
9	0.38	0.076	9	3	13
10	0.13	0.026	20	3	28
11	0.39	0.078	3	4	6
12	0.53	0.106	2	5	5
13	0.79	0.158	1	5	2

$$TOQ_4 = q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_{11}$$

or

$$TOQ_4 = 307 + 20 + 49 + 36 + 6 + 12 + 11 + 6 = 447$$

Table 11 shows the total order quantities for this family of items up to 60 cycle times, in order to compute these values of TOQ_j a computer program was made and it is shown in Appendix B.

Average Inventory for a Family of Items

Figure 7 shows the graph of inventory balances for this fixed cycle system with a cycle time T between orders and a supply lead time τ, which is a random variable that depends on the supplier of the family of items. This graph displays the summation of item quantities q_i in a family where the total demand of all the items per family is given by $D = \Sigma R_i$. The solid line shows the fluctuation of physical inventory on hand and the dashed line the sum of inventory on hand and on order. Note that orders are placed each cycle time T in an amount equal to the total order quantity TOQ_j corresponding to cycle number J. Thus the average total ordering quantity can be calculated as

$$ATOQ = \frac{\sum_{j=1}^{NC} TOQ_j}{NC} \qquad (24)$$

In general the inventory on hand varies from a maximum of $(TOQ_1 + SS)$ at the point where the first order has just been received to a minimum of SS just before the receipt of the following orders.

Table 11. Total Order Quantities for 60 Cycles in Family Example

Cycle Number J	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
TOQ_j	501	441	465	447	425	488	418	447	465	448	418	494	418	441	472
Cycle Number J	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
TOQ_j	447	418	488	418	454	465	441	418	494	425	441	465	447	418	495
Cycle Number J	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
TOQ_j	418	447	465	441	425	494	418	441	465	454	418	488	418	447	472
Cycle Number J	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
TOQ_j	441	418	494	418	448	465	447	418	488	425	447	465	441	418	501

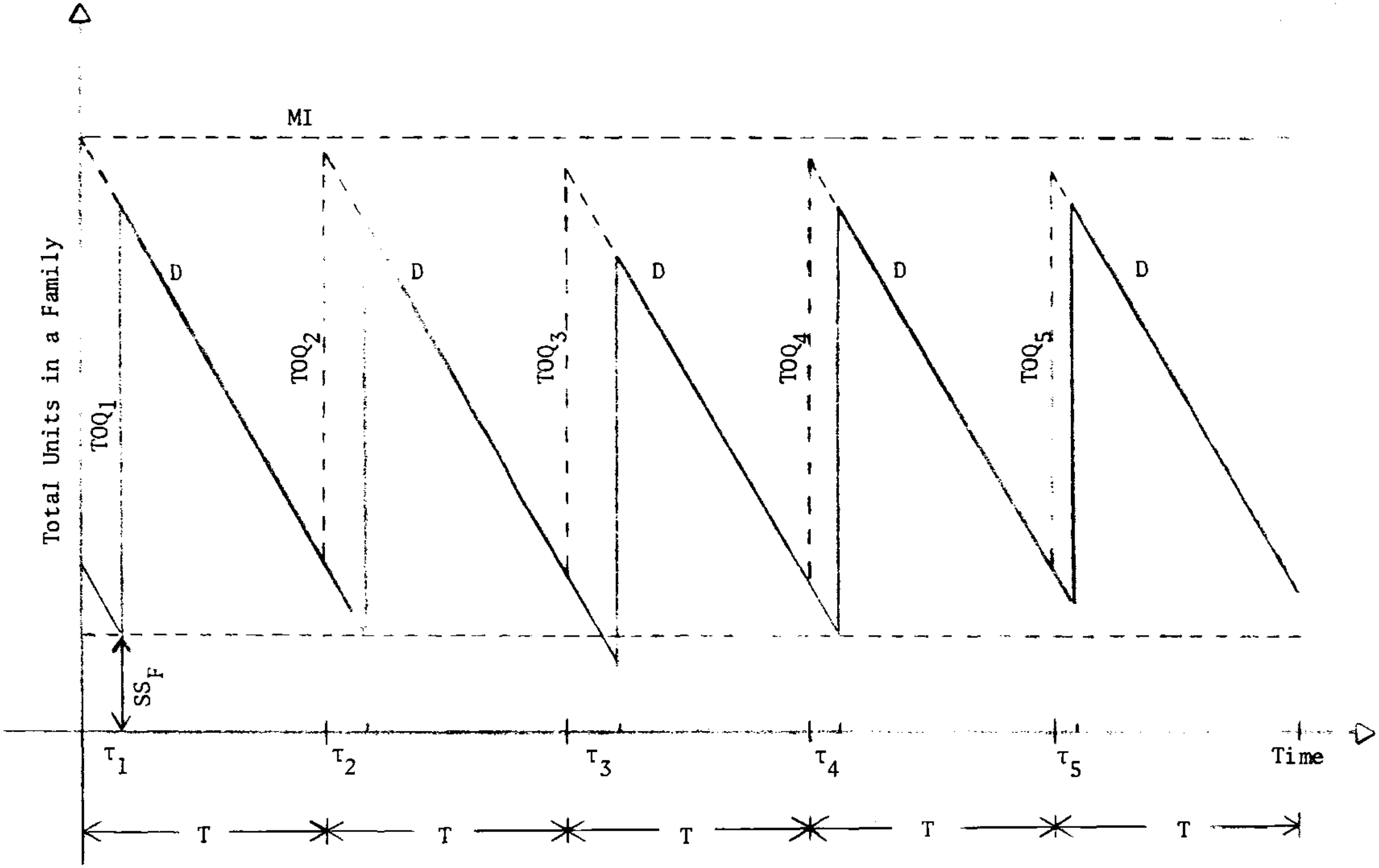


Figure 7. Inventory Balance

From the geometry of Figure 7 the average working stock, that is the inventory on hand not considering the safety stock, is one-half the average total ordering quantity. The average inventory on hand is then this amount added to the family safety stock

$$AI = \frac{1}{2} \frac{\sum^{NC} TOQ_j}{NC} + SS_F \tag{25}$$

At the beginning of any time cycle J, the on hand and on order inventory is the total order quantity TOQ_j plus the family demand times the mean lead time and the safety stock level of the family

$$OHO = TOQ_j + D\tilde{\tau} + SS_F \tag{26}$$

where the maximum inventory on hand and on order occurs at the beginning of the first cycle, or at any cycle which has a $TOQ_j = TOQ_1$

$$MI = TOQ_1 + D\tilde{\tau} + SS_F \tag{27}$$

Illustrative Problem

In order to compute the policy variables T and k_i for the 10 families of the sample data, a computer program was made and it is listed in Appendix C. This program consists of the main program and two subroutines which calculate the average working stock per family and the family safety stocks. The main program also evaluates the total costs of both the current and the joint order inventory models. The values of T, k_i, and q_i are listed in Table 12.

The average working stock was computed using the algorithm developed to calculate the family total order quantity per cycle,

Table 12. Policy Variables as Determined Through the Model

Family	T	Cycle Multiples k_i and Lots Ordered q_i													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.0835	1 83	1 1836	1 175	2 50	1 134	1 184	1 2337	1 250	1 250					
2	0.0824	1 301	1 20	1 48	1 35	2 12	2 12	2 12	3 5	1 41	2 18	4 5	4 3	6 2	
3	0.0820	1 656	1 471	1 102	1 82	2 98	1 297	1 1640	1 410	3 74	1 557				
4	0.1563	1 469	1 1094	4 16	1 23	1 703	1 391	1 43	1 66						
5	0.4217	1 276	1 18	1 44	1 32	1 5	2 11	2 10	2 3	2 8	3 25	4 5	4 3	5 2	
6	0.1027	1 514	4 45	1 1233	2 103	1 334	3 25	4 171							
7	0.1798	1 36	2 7	1 58	1 31	1 144	2 14	2 22	1 99						
8	0.2449	1 1837	1 367	1 551	4 147	1 392	1 294	1 612	1 24	3 168	1 3673	2 50	1 190		
9	0.0941	1 113	1 56	1 19	1 471	1 14	2 12								
10	0.0948	1 569	1 545	1 95	1 24	1 398	1 427	1 142	1 20	1 76	1 9	2 54	2 237	1 123	1 64

TOQ_j, and the first term of Equation (25).

The safety stock computations were accomplished by means of a library program that determines the number of standard deviations from the mean according to the area of stockout probability in the Cumulative Normal Distribution Tables. This number is the value F or the stockout acceptance factor, and F times the standard deviation of the lead time demand distribution gives us the required safety stock that corresponds to the acceptable stockout probability stated by management.

There were two computer runs made. The first calculated the lot sizes q_i and cycle time T per family in order to know the average working family stocks and the number of deliveries per year. Once these values were known, the item safety stock categories were determined according to the management policies of permissible stockouts. In the second run the parameters of every family lead time demand distribution were calculated using Table 4, the lead time parameters and the total demand D per family, then the safety stocks per family were computed and added to the average family working stocks to obtain the average inventory on hand per family.

Analysis of Results

All results are based upon analysis of 60 cycles equivalent to 60 T years. This represents an average of nine years for the 10 families.

The new procurement policy shows a considerable cost reduction--54% for all part numbers--and it points to great potential for improvement. Another important comparison was a decrease of average on hand inventory by 61% for the sample. This was caused by a decrease of 39% on the average working stock and a decrease of 22% in the safety stock. The reduction in working stocks is significant even if the safety stocks were not changed. Thus, there exists sufficient improvement opportunities over the current system to warrant the cost of developing and installing this new fixed cycle system. Table 13 illustrates these comparisons and the magnitude of the savings.

The strategy selected for making the inventory control decisions --the cycle time T and cycle multiples k_i--depend upon the costs that are influenced by these decisions. These costs, in turn, depend upon the type of system within which the inventory is functioning. The inventory model and decisions rules presented in this thesis are suitable for joint ordering families of items from common suppliers. Decisions regarding replenishment of a group of items do not influence costs elsewhere in the system. The results also depend in large measure on other components and on the characteristics of the total control system established for making the decisions.

Using the sample data, the convergence of the iterative procedure was usually quite rapid. The maximum number of iterations needed to converge was five, and the average was three.

All the k_i values for every iteration per family were printed in order to have a numerical argument for the illustration of the convergence. These results are shown in Appendix D (Computer Output for Sample Problem), where the values of the family policy variables are printed, together with the variables of the current procurement inventory policies.

Table 13. Comparison of Procurement Policies

Family	Average Working Stock	Safety Stock	Average On-hand Inventory	Total Cost
1	2,593	1,963	4,556	321.14
2	249	295	544	320.20
3	2,137	2,246	4,383	330.14
4	1,373	461	1,834	165.05
5	214	13	227	62.67
6	1,090	545	1,635	228.47
7	194	41	235	140.13
8	3,900	646	4,546	114.61
9	223	210	433	259.28
10	1,346	966	2,312	320.74
New Procurement Policy	13,319	7,386	20,705	2,262.43
Current Procurement Policy	33,810	19,299	53,109	4,961.26

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

An inventory model was used to understand the behavior of inventories under a condition of joint order quantities for a group of items from a common source. Variables and parameters were isolated in a practical way to deal with the total inventory control problem. One of the significant problems involved in this application is the adaption of the model for use with a large number of inventory items. Here, modern data processing methods can rescue for practical use what might otherwise be a model of only theoretical interest. It is possible to learn more about inventory in trying to create the records than in analyzing them.

In cases where a large number of items are regularly ordered from common suppliers, it appears worthwhile to abandon an order point system and place orders for grouped items by some regular schedule in order to take advantage of the savings involved.

The key difference in the fixed reorder system compared to the fixed order quantity system is that action is triggered periodically rather than by an order point. The quantity ordered, however, varies depending on the cycle multiples k_i of the family of items. Thus, with the fixed reorder cycle system orders of varying size are placed in a fixed periodic cycle whereas with the fixed reorder quantity system we place orders of fixed size by a variable time cycle.

Proper data are difficult to recognize and to find. It is often possible to simplify the data processing problem through classification of whole groups of items whose demand distribution may be adequately described by one of the standard distributions, such as the normal, Poisson, or negative exponential distributions.

Good managerial practice would not give the same time and attention to the control of all items. Classification of items by their total inventory value or otherwise critical parameters may make it possible to establish progressively tighter control over higher valued critical items and relatively simple and loose controls over inexpensive and/or noncritical items.

At every cycle time T it is known how much to order, because the family total order quantity per cycle has already been determined. Thus, there is no extra cost for making the periodic review, except for the cost of maintaining the perpetual inventory records which are used as a basis for review decisions. In cases of demand changes the reorder cycle system normally provides more frequent information. Since the demand rates are under surveillance every review period T, the system responds directly to usage in the immediate past period. Having this quick response to changes in demand, the fixed cycle system is in general applicable for a high-activity situation where close surveillance over both demand and inventory level is of importance.

Recommendations for Further Study

It would be helpful to have an analysis on the association of all the cycle times T that are approximately equal, so that these families could be ordered together in the same period of time T and examine the effects on the total costs.

It would be worthwhile to examine the situation where the holding rate H is not the same for all items and to determine the cycle multiples k_i with the effects on aggregate inventory. There is also the problem of allocation, where a decision must be made on the quantities for each item in the group that will accumulate the desired total and at the same time balance the inventory in some sense.

Improvements can be made by considering changes in demand and examining the necessary approaches for the adoption of this joint ordering inventory model. Additional improvements may be available in the analysis of the economic consequences of ordering rules that take account of the status of either many items at one location, or the same item in many locations. These problems can quickly get to a degree of complexity that is unmanageable. Complete answers to these problems appear to be beyond the state of the art as shown in the literature.

APPENDICES

APPENDIX A

TOTAL COST DEVIATION PROGRAM

The program used for calculating the total cost deviation from the minimum total yearly cost is listed below in FORTRAN IV Language

```
C     LUIS PATRON
      DIMENSION A(20),A1(20),R(20),UC(20),AR(20),RK(20)
      DIMENSION K(15,20),F(30),TC(30),CD(30),R1(20)
      DATA K/300*0/
      READ 10, N,CO1,HR
   10 FORMAT(20X,I3,F7.2,F5.2)
      READ 15, (UC(I),I=1,N)
      READ 15, (AR(I),I=1,N)
   15 FORMAT(8F10.2)
      READ 20, (F(L),L=1,28)
   20 FORMAT(20F4.2)
      DO 5 I=1,N
      A1(I)=0.50
    5 R1(I)=HR*UC(I)
      NP=3
      DO 450 LK=1,NP
      DO 500 L=1,28
      DO 6 I=1,N
    6 K(1,1)=1
      IF(LK.EQ.1) GO TO 550
      IF(LK.EQ.2) GO TO 600
      IF(LK.EQ.3) GO TO 650
      GO TO 1600
  550 CO=CO1*F(L)
      DO 555 I=1,N
      R(I)=R1(I)
  555 A(I)=A1(I)
      GO TO 1500
  600 CO=CO1
      DO 660 I=1,N
      R(I)=R1(I)
  660 A(I)=A1(I)*F(L)
      GO TO 1500
  650 CO=CO1
```

```
      DO 670 I=1,N
      A(I)=A1(I)
  670 R(I)=R1(I)*F(L)
 1500 DO 90 J=1,15
      IF(J.EQ.1) GO TO 96
      J1=J-1
      II=0
      DO 95 I=1,N
      IF(K(J,I).EQ.K(J1,I)) II=II+1
   95 CONTINUE
      IF(II.EQ.N) GO TO 750
   96 SAK=0
      TD=0
      DO 100 I=1,N
      RCM=K(J,I)
      SAK=SAK+A(I)/RCM
  100 TD=TD+R(I)*RCM*AR(I)
      TN=2.*(CO+SAK)
      TS=TN/TD
      T=SQRT(TS)
      J2=J+1
      DO 110 I=1,N
      SK=2.*A(I)/(R(I)*AR(I))
      SQK=SQRT(SK)
      RK(I)=SQK/T
      IF(RK(I).GE.0.AND.RK(I).LT.1.414) GO TO 30
      IF(RK(I).GE.1.414.AND.RK(I).LT.2.449) GO TO 35
      IF(RK(I).GE.2.449.AND.RK(I).LT.3.464) GO TO 40
      IF(RK(I).GE.3.464.AND.RK(I).LT.4.472) GO TO 45
      IF(RK(I).GE.4.472.AND.RK(I).LT.5.477) GO TO 50
      IF(RK(1).GE.5.477.AND.RK(I).LT.6.480) GO TO 55
      K(J2,I)=RK(I)+0.52
      GO TO 110
   30 K(J2,I)=1
      GO TO 110
   35 K(J2,I)=2
      GO TO 110
   40 K(J2,I)=3
      GO TO 110
   45 K(J2,I)=4
      GO TO 110
   50 K(J2,I)=5
      GO TO 110
   55 K(J2,I)=6
  110 CONTINUE
   90 CONTINUE
      PRINT 2000
 2000 FORMAT(10X,'NO CONVERGENCE IN 15 ITERATIONS')
  750 IF(L.EQ.1) GO TO 128
      SAKL=0
      SRKT=0
```

```
      DO 115 I=1,N
      RCM=K(J,I)
      SAKL=SAKL+A1(I)/RCM
  115 SRKT=SRKT+R1(I)*RCM*AR(I)
      SAKL=CO1+SAKL
      SRKT=SRKT*T
      TC1=SAKL/T
      TC2=SRKT/2.
      TC(L)=TC1+TC2
      CD(L)=((TC(L)-TC(1))/TC(1))*100.
      GO TO 520
  128 TC(1)=(CO+SAK)/T+T*TO/2.
      CD(1)=0
  520 PRINT 1000, LK,L,F(L),TC(L),CD(L)
 1000 FORMAT(10X,2I5,3F10.2/)
  500 CONTINUE
      GO TO 450
 1600 PRINT 1650, LK
 1650 FORMAT(10X,I3,' PARAMETERS REQUIRED')
  450 CONTINUE
      END
```

APPENDIX B

ORDER QUANTITY PROGRAM

The program used to compute the family total order quantities for every cycle is listed below in FORTRAN IV Language

```
      DIMENSION K(25),Q(25),TOQ(60),KJL(60,60),P(25,60)
      READ 1, N,NC,KG
    1 FORMAT(3I5)
      READ 2, (K(I),I=1,N)
    2 FORMAT(40I2)
      READ 3, (Q(I),I=1,N)
    3 FORMAT(10F8.2)
      SQ1=0
      DO 100 I=1,N
  100 SQ1=SQ1+Q(I)
      TOQ(1)=SQ1
      DO 101 J=2,NC
      SQ=0
      J1=J+1
      DO 105 M=1,J
      L=J1-M
      RJ=J
      RL=L
      RJL=RJ/RL
      IJL=RJL
      DRI=RJL-IJL
      IF(DRI.GT.0) GO TO 104
      KJL(J,L)=L
      GO TO 105
  104 KJL(J,L)=0
  105 CONTINUE
  106 DO 110 I=1,N
      P(I,J)=0
      DO 115 L=1,KG
  115 IF(K(I).EQ.KJL(J,L)) GO TO 109
      GO TO 110
  109 P(I,J)=1.
  110 SQ=SQ+Q(I)*P(I,J)
  101 TOQ(J)=SQ
      PRINT 200, (TOQ(J),J=1,NC)
  200 FORMAT(5X,12F8.2/)
      END
```

APPENDIX C

INVENTORY SYSTEM EVALUATION

Main Program

A computer model based upon this system was prepared utilizing FORTRAN IV, and the computer output furnished measures of the system's performance through cumulative costs for ordering, holding stock and the average on-hand inventory.

```
C       LUIS PATRON
C       MAIN PROGRAM
C       JOINT ORDERING INVENTORY MODEL
C
        COMMON/KQN/K(15,110),QN(110),I1,I2,JC,KG,NC,WS
        DIMENSION SD(10),ALT(10),A(110),R(110),RK(110),QC(110)
        DIMENSION AR(110),UC(110),IV(110),ICAT(110),PP(12,4)
        READ 5, INT,CO,HR
      5 FORMAT(I4,F7.2,F5.2)
        READ 6, (ALT(NF),NF=1,10)
        READ 6, (SD(NF),NF=1,10)
      6 FORMAT(10F8.2)
        READ 7, ((PP(IP,JP),IP=1,12),JP=1,4)
      7 FORMAT(12F6.4)
        READ 8, (ICAT(IN),IN=1,INT)
      8 FORMAT(40I2)
        NF=1
        NIF=0
        TTC=0
        TCQC=0
        I1=1
        I2=0
        WSC=0
        SS1=0
        DO 10 IN=1,INT
        READ 15, AR(IN),UC(IN),IV(IN)
     15 FORMAT(4X,F10.2,F8.2,I2)
        A(IN)=0.40
        R(IN)=HR*UC(IN)
```

```
C     COMPUTATIONS OF CURRENT INVENTORY POLICY VARIABLES
      A1=(2.*AR(IN)*CO)/R(IN)
      QC(IN)=SQRT(A1)
      WSC=WSC+QC(IN)
      SS1=SS1+AR(IN)/12.
      B1=2.*AR(IN)*CO*R(IN)
      CQC=SQRT(B1)
      TCQC=TCQC+CQC
      IF(IN.EQ.INT) GO TO 25
      IF(IV(IN).EQ.NF) GO TO 20
      N=NIF
      GO TO 26
   25 N=NIF+1
   26 NIF=1
      I2=I2+N
C
C     COMPUTATIONS OF NEW INVENTORY POLICY VARIABLES
C     CYCLE TIME T AND CYCLE MULTIPLE K(I)
      DO 85 I=I1,I2
   85 K(1,I)=1
      DO 90 J=1,15
      IF(J.EQ.1) GO TO 96
      J1=J-1
      II=0
      DO 95 I=I1,I2
      IF(K(J,I).EQ.K(J1,I)) II=II+1
   95 CONTINUE
      IF(II.EQ.N) GO TO 750
   96 SAK=0
      TD=0
      DO 100 I=I1,I2
      RCM=K(J,I)
      SAK=SAK+A(I)/RCM
  100 TD=TD+R(I)*RCM*AR(I)
      TN=2.*(CO+SAK)
      TS=TN/TD
      T=SQRT(TS)
      J2=J+1
      DO 110 I=I1,I2
      SK=2.*A(I)/(R(I)*AR(I))
      SQK=SQRT(SK)
      RK(I)=SQK/T
      IF(RK(I).GE.0.AND.RK(I).LT.1.414) GO TO 30
      IF(RK(I).GE.1.414.AND.RK(I).LT.2.449) GO TO 35
      IF(RK(I).GE.2.449.AND.RK(I).LT.3.464) GO TO 40
      IF(RK(I).GE.3.464.AND.RK(I).LT.4.472) GO TO 45
      IF(RK(I).GE.4.472.AND.RK(I).LT.5.477) GO TO 50
      IF(RK(I).GE.5.477.AND.RK(I).LT.6.480) GO TO 55
      K(J2,I)=RK(I)+0.52
      GO TO 110
```

```
   30 K(J2,I)=1
      GO TO 110
   35 K(J2,I)=2
      GO TO 110
   40 K(J2,I)=3
      GO TO 110
   45 K(J2,I)=4
      GO TO 110
   50 K(J2,I)=5
      GO TO 110
   55 K(J2,I)=6
  110 CONTINUE
   90 CONTINUE
      PRINT 2000
 2000 FORMAT(10X,'NO CONVERGENCE IN 15 ITERATIONS')
  750 TC=(CO+SAK)/T+T*TD/2.
      JC=J
      PRINT 751
  751 FORMAT(1H1,10X,'K(I) VALUES FOR ALL ITERATIONS'/)
      DO 752 J=1,JC
      PRINT 753, (K(J,I),I=I1,I2)
  753 FORMAT(10X,15I5)
  752 CONTINUE
      PRINT 754, JC
  754 FORMAT(/,10X,'CONVERGENCE IN',I5,' ITERATIONS'//)
C
C     DETERMINING THE GRETATEST CYCLE MULTIPLE KG
C     AND THE LOTS Q(I) PER FAMILY
      KG=0
      DO 755 I=I1,I2
      IF(KG.LT.K(JC,I)) KG=K(JC,I)
  755 QN(I)=K(JC,I)*T*AR(I)
      PRINT 760, T,N,NF
  760 FORMAT(//,10X,'T = ',F8.4,11X,'FAMILY ITEMS = ',I3,'
     1FAMILY NUMBER = ',I2/)
      PRINT 765, (K(JC,I),I=I1,I2)
  765 FORMAT(10X,'K(I)',14I5)
      PRINT 770, (QN(I),I=I1,I2)
  770 FORMAT(10X,'Q(I)',7F10.0)
      PRINT 775, TC
  775 FORMAT(/,10X,'TOTAL COST = ',F10.2)
C
C     AVERAGE WORKING STOCK PER FAMILY
      NC=60
      CALL TOQC
      IWS=WS
      WS=IWS
      PRINT 801, WS
  801 FORMAT(/,14X,'AVERAGE WORKING STOCK = ',F15.0)
      PRINT 802
  802 FORMAT(/,14X,'DELIVERIES',17X,'SAFETY STOCK')
```

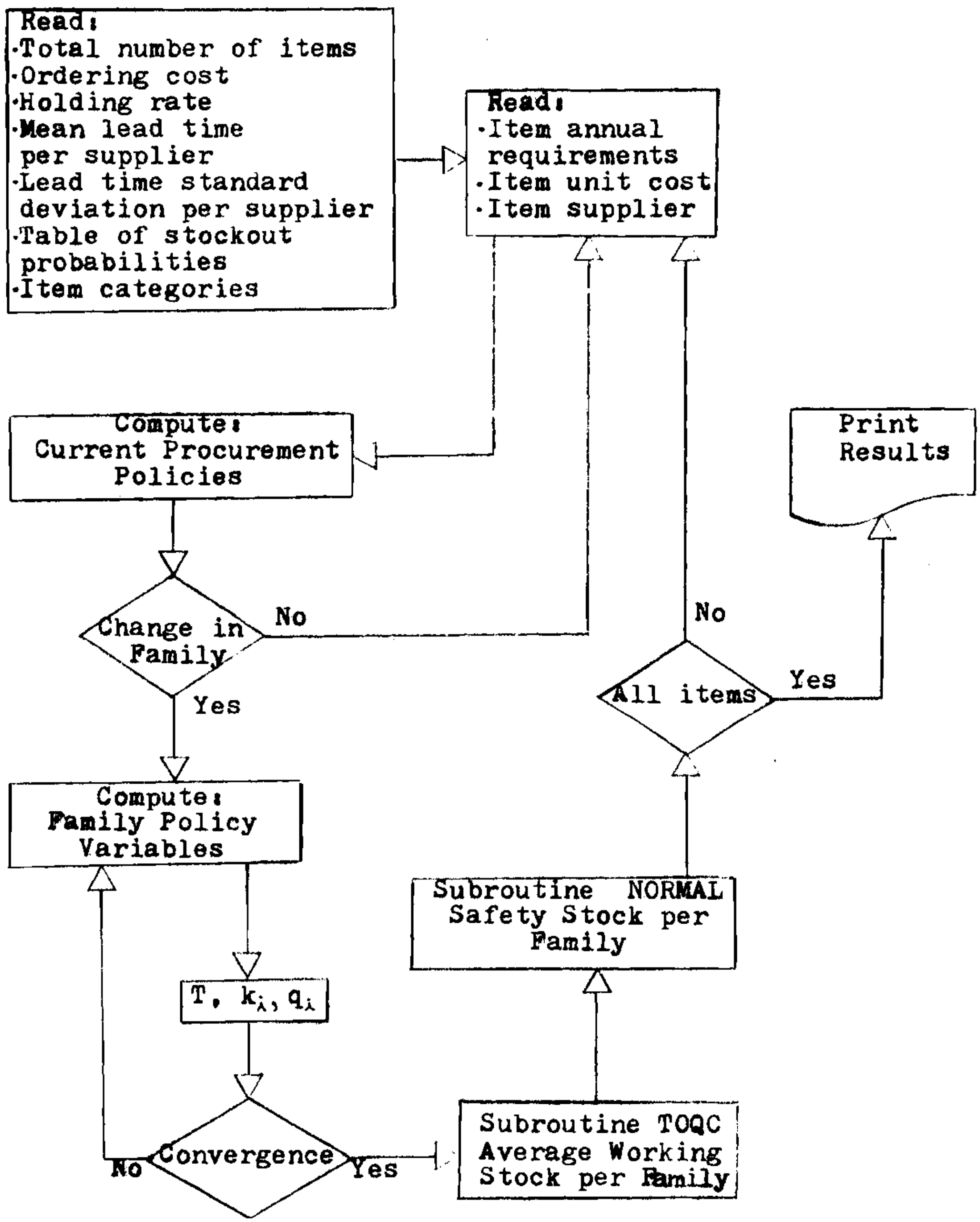
```
C
C     SAFETY STOCK PER FAMILY AND AVERAGE INVENTORY ON HAND
      SSF=0
      DO 803 I=I1,I2
      DEL=1./(K(JC,I)*T)
      IP=DEL
      JP=ICAT(I)
      SD1=(AR(I)/52.)*SD(NF)
      RPS=1.-PP(IP,JP)
      CALL NORMAL(RPS,SD1,0.0,SS,Z)
      ISS=SS
      SS=ISS
      PRINT 804, IP,SS
  804 FORMAT(17X,I4,17X,F15.0)
      SSF=SSF+SS
  803 CONTINUE
      PRINT 805, SSF
  805 FORMAT(/,16X,'FAMILY SAFETY STOCK = ',F15.0)
      AIN=WS+SSF
      PRINT 806, AIN
  806 FORMAT(/,10X,'AVERAGE INVENTORY ON HAND = ',F15.0)
      TTC=TTC+TC
      I1=I1+N
      NF=NF+1
      GO TO 10
   20 NIF=NIF+1
   10 CONTINUE
      PRINT 776, TTC
  776 FORMAT(//,49X,'SUM OF TOTAL COSTS = ',F12.2)
C
C     PRINTING OF CURRENT INVENTORY POLICIES
      PRINT 780, INT
  780 FORMAT(1H1,10X,'TOTAL NUMBER OF ITEMS = ',I4/)
      PRINT 785, (QC(IN),IN=1,INT)
  785 FORMAT(10X,'Q(I)',7F10.0)
      PRINT 790, TCQC
  790 FORMAT(/,10X,'TOTAL COST = ',F10.2)
      WSC=WSC/2.
      PRINT 900, WSC
  900 FORMAT(/,14X,'AVERAGE WORKING STOCK = ',F15.0)
      PRINT 901, SS1
  901 FORMAT(/,23X,'SAFETY STOCK = ',F15.0)
      AIC=WSC+SS1
      PRINT 902, AIC
  902 FORMAT(/,10X,'AVERAGE INVENTORY ON HAND = ',F15.0)
      DTC=TCQC-TTC
      PRINT 795, DTC
  795 FORMAT(////,40X,'*** SAVINGS = ',F10.2,2X,'***')
      END
```

Subroutine to Compute the Average Working Stock

This subroutine is based on the total family ordering quantity per cycle algorithm, and computes the average working stock per family according to Equation (24)

```
      SUBROUTINE TOQC
      COMMON/KQN/K(15,110),QN(110),I1,I2,JC,KG,NC,WS
      DIMENSION QT(25),KJL(60,60),P(25,60)
      I=0
      DO 90 IT=I1,I2
      I=I+1
   90 QT(I)=QN(IT)
      I3=I2-I1+1
      SQ1=0
      DO 100 I=1,I3
  100 SQ1=SQ1+QT(I)
      WS=0
      DO 101 J=2,NC
      SQ=0
      J1=J+1
      DO 105 M=1,J
      L=J1-M
      RJ=J
      RL=L
      RJL=RJ/RL
      IJL=RJL
      DRI=RJL-IJL
      IF(DRI.GT.0) GO TO 104
      KJL(J,L)=L
      GO TO 105
  104 KJL(J,L)=0
  105 CONTINUE
      DO 110 I=1,I3
      P(I,J)=0
      DO 115 L=1,KG
  115 IF(K(JC,I).EQ.KJL(J,L)) GO TO 109
      GO TO 110
  109 P(I,J)=1.
  110 SQ=SQ+QT(I)*P(I,J)
  101 WS=WS+SQ
      RNC=NC
      WS=0.5*(WS/RNC)
      RETURN
      END
```

Flow Chart of Main Program



Glossary of Terms

1.	INT	Total number of items.
2.	CO	Ordering cost.
3.	HR	Holding rate.
4.	NF	Family number.
5.	NIF	Number of items per family.
6.	ALT(NF)	Mean lead time per family.
7.	SD(NF)	Family lead time standard deviation.
8.	ICAT(I)	Item category, I is the item code or number.
9.	A(I)	Marginal cost per item.
10.	AR(I)	Annual requirement per item.
11.	UC(I)	Item unit cost.
12.	IV(I)	Item vendor.
13.	R(I)	Carrying charge per item.
14.	RK(I)	Real value of cycle multiple k_i.
15.	K(J,I)	Rounded integer value of cycle multiple k_i, J is the iteration number of the iterative procedure.
16.	T	Cycle time T.
17.	TC	Total annual cost.
18.	QN(I)	Lot q_i.
19.	WS	Average working stock.
20.	PP(IP,JP)	Table of stockout probabilities, IP is the number of deliveries $(1/k_i T)$ and JP is the item category.
21.	SD1	Lead time demand standard deviation.
22.	SSF	Safety stock per family.
23.	AIN	Average inventory on hand.

Sample Problem Data

(Number of items,Ordering cost,Holding rate)

100 10 24

(Mean lead time) 3 4 4 2 350 275 450 375 280 4
(Standard deviation) 175 2 225 141 166 187 202 180 176 228

Table of stockout probability	5000	3333	2500	2000	1666	1428	1250	1111	1000	0909	0833	0769
		5000	4000	3333	2857	2500	2222	2000	1818	1666	1533	1428
			5000	4285	3757	3333	3000	2727	2500	2307	2142	2000
				5000	4444	4000	3636	3333	3076	2857	2666	2500

Item category
1 2 2 1 2 2 3 2 2 1 1 1 1 1 1 2 2 2 3 3 3 3 3 3 3 3 2 1 2 3 2 2 1 1 1 1 2 1 3 1
1 1 1 1 2 2 2 2 2 2 2 2 2 2 3 3 2 4 4 2 1 4 2 3 3 2 1 2 1 2 3 3 2 1 2 1 3 2 2 1
3 4 3 2 3 4 3 2 2 4 2 3 2 3 3 2 4 4 4 3

(Item number,Annual requirements,Item cost,Family number)

1	1000	24	1	26	1000	94	3	51	3	39	5	76	100	40	8
2	22000	24	1	27	600	20	3	52	2	53	5	77	228	04	8
3	2100	22	1	28	3625	65	3	53	1	79	5	78	15000	02	8
4	300	28	1	29	20000	05	3	54	5000	38	6	79	102	15	8
5	1600	27	1	30	5000	13	3	55	110	20	6	80	775	30	8
6	2200	31	1	31	300	15	3	56	12000	40	6	81	1200	450	9
7	28000	25	1	32	6800	40	3	57	500	16	6	82	600	325	9
8	3000	32	1	33	3000	60	4	58	3250	65	6	83	200	175	9
9	3000	27	1	34	7000	10	4	59	80	45	6	84	5000	56	9
10	3650	272	2	35	25	35	4	60	415	06	6	85	150	415	9
11	240	266	2	36	150	120	4	61	200	250	7	86	65	275	9
12	580	226	2	37	4500	15	4	62	20	115	7	87	6000	25	10
13	430	225	2	38	2500	30	4	63	325	45	7	88	5750	15	10
14	75	294	2	39	275	50	4	64	175	30	7	89	1000	100	10
15	75	253	2	40	420	29	4	65	800	175	7	90	253	240	10
16	70	254	2	41	654	72	5	66	40	50	7	91	4200	75	10
17	20	291	2	42	43	66	5	67	60	60	7	92	4500	50	10
18	500	238	2	43	104	26	5	68	550	180	7	93	1500	45	10
19	110	213	2	44	77	25	5	69	7500	08	8	94	210	460	10
20	15	239	2	45	13	94	5	70	1500	10	8	95	800	100	10
21	10	253	2	46	13	53	5	71	2250	05	8	96	100	220	10
22	5	279	2	47	12	54	5	72	150	03	8	97	285	30	10
23	8000	36	3	48	4	91	5	73	1600	09	8	98	1250	10	10
24	5750	86	3	49	9	38	5	74	1200	10	8	99	1300	50	10
25	1250	73	3	50	20	13	5	75	2500	07	8	100	670	150	10

APPENDIX D

COMPUTER OUTPUT FOR SAMPLE PROBLEM

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1	1	1
1	1	1	2	1	1	1	1	1
1	1	1	2	1	1	1	1	1

CONVERGENCE IN 3 ITERATIONS

T = .0835 FAMILY ITEMS = 9 FAMILY NUMBER = 1

K(I) 1 1 1 2 1 1 1 1 1

Q(I) 83. 1836. 175. 50. 134. 184. 2337.

Q(I) 250. 250.

TOTAL COST = 321.14

AVERAGE WORKING STOCK = 2593.

DELIVERIES	SAFETY STOCK
11	46.
11	755.
11	72.
5	9.
11	54.
11	75.
11	746.
11	103.
11	103.

FAMILY SAFETY STOCK = 1963.

AVERAGE INVENTORY ON HAND = 4556.

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	2	2	3	1	1	3	4	5
1	1	1	1	2	2	2	3	1	1	4	4	6
1	1	1	1	2	2	2	3	1	2	4	4	6
1	1	1	1	2	2	2	3	1	2	4	4	6

CONVERGENCE IN 5 ITERATIONS

```
T =     .0824            FAMILY ITEMS =  13               FAMILY NUMBER =  2

K(I)    1     1     1     1     2     2     2     3     1     2     4     4     6
Q(I)       301.        20.         48.         35.         12.         12.         12.
Q(I)         5.        41.         18.          5.          3.          2.
```

TOTAL COST = 320.20

AVERAGE WORKING STOCK = 249.

DELIVERIES	SAFETY STOCK
12	200.
12	13.
12	31.
12	23.
6	3.
6	3.
6	1.
4	0.
12	20.
6	1.
3	0.
3	0.
2	0.

FAMILY SAFETY STOCK = 295.

AVERAGE INVENTORY ON HAND = 544.

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1	1	1	1
1	1	1	1	2	1	1	1	3	1
1	1	1	1	2	1	1	1	3	1

CONVERGENCE IN 3 ITERATIONS

T = .0820 FAMILY ITEMS = 10 FAMILY NUMBER = 3

K(I)	1	1	1	1	2	1	1	1	3	1

Q(I)	656.	471.	102.	82.	98.	297.	1640.
Q(I)	410.	74.	557.				

TOTAL COST = 330.14

AVERAGE WORKING STOCK = 2137.

DELIVERIES	SAFETY STOCK
12	291.
12	209.
12	45.
12	36.
6	17.
12	223.
12	924.
12	182.
4	5.
12	314.

FAMILY SAFETY STOCK = 2246.

AVERAGE INVENTORY ON HAND = 4383.

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1	1
1	1	4	1	1	1	1	1
1	1	4	1	1	1	1	1

CONVERGENCE IN 3 ITERATIONS

T = .1563 FAMILY ITEMS = 8 FAMILY NUMBER = 4

K(I)	1	1	4	1	1	1	1
Q(I)	469.	1094.	16.	23.	703.	391.	43.
Q(I)	66.						

TOTAL COST = 165.05

AVERAGE WORKING STOCK = 1373.

DELIVERIES	SAFETY STOCK
6	86.
6	202.
1	0.
6	4.
6	82.
6	72.
6	3.
6	12.

FAMILY SAFETY STOCK = 461.

AVERAGE INVENTORY ON HAND = 1834.

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	2	2	2	2	2	4	4	4
1	1	1	1	1	2	2	2	2	3	4	4	5
1	1	1	1	1	2	2	2	2	3	4	4	5

CONVERGENCE IN 4 ITERATIONS

T = .4217 FAMILY ITEMS = 13 FAMILY NUMBER = 5

K(I)	1	1	1	1	1	2	2	2	2	3	4	4	5
Q(I)	276.		18.		44.		32.		5.		11.		10.
Q(I)	3.		8.		25.		5.		3.		2.		

TOTAL COST = 62.67

AVERAGE WORKING STOCK = 214.

DELIVERIES	SAFETY STOCK
2	8.
2	0.
2	1.
2	1.
2	0.
1	1.
1	1.
1	0.
1	1.
0	0.
0	0.
0	0.
0	0.

FAMILY SAFETY STOCK = 13.

AVERAGE INVENTORY ON HAND = 227.

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1
1	4	1	2	1	3	3
1	4	1	2	1	3	4
1	4	1	2	1	3	4

CONVERGENCE IN 4 ITERATIONS

T = .1027 FAMILY ITEMS = 7 FAMILY NUMBER = 6

K(I)	1	4	1	2	1	3	4
Q(I)	514.	45.	1233.	103.	334.	25.	171.

TOTAL COST = 228.47

AVERAGE WORKING STOCK = 1090.

DELIVERIES	SAFETY STOCK
9	163.
2	15.
9	291.
4	7.
9	58.
3	11.
2	0.

FAMILY SAFETY STOCK = 545.

AVERAGE INVENTORY ON HAND = 1635.

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1	1
1	2	1	1	1	2	2	1
1	2	1	1	1	2	2	1

CONVERGENCE IN 3 ITERATIONS

T = .1798 FAMILY ITEMS = 8 FAMILY NUMBER = 7

K(I)	1 2	1 1	1 2	2 1			
Q(I)	36.	7.	58.	31.	144.	14.	22.
Q(I)	99.						

TOTAL COST = 140.13

AVERAGE WORKING STOCK = 194.

DELIVERIES	SAFETY STOCK
5	7.
2	3.
5	7.
5	2.
5	9.
2	0.
2	1.
5	12.

FAMILY SAFETY STOCK = 41.

AVERAGE INVENTORY ON HAND = 235.

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	3	1	1	1	1	2	1	2	1
1	1	1	4	1	1	1	1	3	1	2	1
1	1	1	4	1	1	1	1	3	1	2	1

CONVERGENCE IN 4 ITERATIONS

```
T =     .2449            FAMILY ITEMS =  12            FAMILY NUMBER =  8

K(I)    1     1     1     4     1     1     1     1     3     1     2     1
Q(I)     1837.       367.        551.        147.        392.        294.        612.
Q(I)       24.       168.       3673.         50.        190.

TOTAL COST =      114.61

    AVERAGE WORKING STOCK =            3900.

    DELIVERIES                   SAFETY STOCK
        4                             218.
        4                              22.
        4                              14.
        1                              20.
        4                              23.
        4                              34.
        4                              37.
        4                               2.
        1                              31.
        4                             223.
        2                               0.
        4                              22.

      FAMILY SAFETY STOCK =             646.

AVERAGE INVENTORY ON HAND =            4546.
```

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1
1	1	1	1	1	2
1	1	1	1	1	2

CONVERGENCE IN 3 ITERATIONS

T = .0941 FAMILY ITEMS = 6 FAMILY NUMBER = 9

K(I)	1	1	1	1	1	2
Q(I)	113.	56.	19.	471.	14.	12.

TOTAL COST = 259.28

AVERAGE WORKING STOCK = 223.

DELIVERIES	SAFETY STOCK
10	29.
10	11.
10	4.
10	163.
10	3.
5	0.

FAMILY SAFETY STOCK = 210.

AVERAGE INVENTORY ON HAND = 433.

K(I) VALUES FOR ALL ITERATIONS

1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	2	2	1	1
1	1	1	1	1	1	1	1	1	1	2	2	1	1

CONVERGENCE IN 3 ITERATIONS

T = .0948 FAMILY ITEMS = 14 FAMILY NUMBER = 10

K(I)	1	1	1	1	1	1	1	1	1	1	2	2	1	1
Q(I)		569.		545.		95.		24.		398.		427.		142.
Q(I)		20.		76.		9.		54.		237.		123.		64.

TOTAL COST = 320.73

AVERAGE WORKING STOCK = 1346.

DELIVERIES	SAFETY STOCK
10	193.
10	243.
10	42.
10	6.
10	178.
10	145.
10	63.
10	6.
10	25.
10	4.
5	1.
5	7.
10	32.
10	21.

FAMILY SAFETY STOCK = 966.

AVERAGE INVENTORY ON HAND = 2312.

SUM OF TOTAL COSTS = 2262.43

TOTAL NUMBER OF ITEMS = 100

Q(I)	589.	2764.	892.	299.	703.	769.	3055.
Q(I)	884.	962.	334.	87.	146.	126.	46.
Q(I)	50.	48.	24.	132.	66.	23.	18.
Q(I)	12.	1361.	746.	378.	298.	500.	682.
Q(I)	5774.	1790.	408.	1190.	645.	2415.	77.
Q(I)	102.	1581.	833.	214.	347.	275.	74.
Q(I)	183.	160.	34.	45.	43.	19.	44.
Q(I)	113.	25.	18.	10.	1047.	214.	1581.
Q(I)	510.	645.	122.	759.	82.	38.	245.
Q(I)	220.	195.	82.	91.	160.	2795.	1118.
Q(I)	1936.	645.	1217.	1000.	1725.	144.	689.
Q(I)	7006.	238.	464.	149.	124.	98.	863.
Q(I)	55.	44.	1414.	1787.	289.	93.	683.
Q(I)	866.	527.	62.	258.	62.	281.	1021.
Q(I)	465.	193.					

TOTAL COST = 4961.26

AVERAGE WORKING STOCK = 33810.

SAFETY STOCK = 19299.

AVERAGE INVENTORY ON HAND = 53109.

*** SAVINGS = 2698.84 ***

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